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LETTER TO THE EDITORS

DERIVATION OF THE IMPULSE RESPONSE: COMMENTS ON THE METHOD OF ROUFS AND BLOMMAERT

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Our understanding of visual responses in the time domain has been greatly improved by the application of linear systems theory. This approach treats the eye as a linear filter whose input is the temporal waveform of the stimulus and whose output is a temporal response at some unspecified site within the observer. Psychophysical responses depend upon this internal response; a typical assumption is that the observer responds "yes, I see it" when an excursion of the internal response just meets some criterion. The great value of the linear approach is its power to simplify. If the response to a very brief pulse (the *impulse response*) is known, then the response to (and hence the visibility of) any input waveform whatever can be predicted. Though the importance of the impulse response is widely recognized, a method of deriving this function from empirical data has not yet been developed.

The significance and elusiveness of the impulse response lend particular importance to a recent paper by Roufs and Blommaert (1981), in which the impulse response is derived by a new method. The paper also merits attention because the impulse response derived is unusual, as shown in Fig. 1. It has three phases of alternating sign, the middle one being the largest. This contrasts with more conventional impulse responses, like that in the inset to Fig. 2, which typically have only two substantial phases, the first being largest. In this note, I question two of the assumptions made by Roufs and Blommaert, and show that their data are consistent with an impulse response of the conventional sort shown in the inset to Fig. 2.

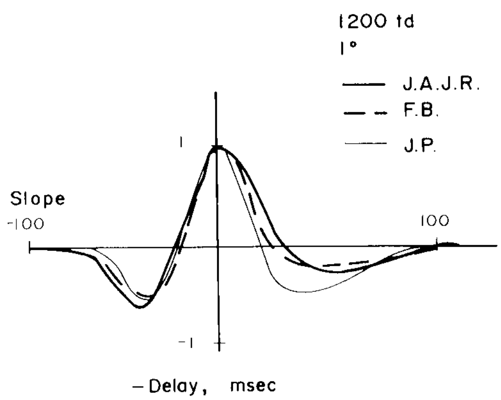

Fig. 1. Slope vs delay functions of three observers estimated by Roufs and Blommaert (1981). Under their assumptions (1) and (2) described above, this function may be interpreted as the impulse response of the underlying linear filter. Note that the abscissa indicates minus delay.

Roufs and Blommaert measured thresholds for combinations of a "probe" flash and a "test" flash, both of 2 msec duration. The test was delayed relative to the probe by various amounts, and the amplitude of the test flash was never more than 30% of the amplitude of the probe. At each delay, they estimated from these thresholds the slope of the relation between sensitivity and the ratio of amplitudes of probe and test. These slope vs delay curves are shown for three observers in Fig. 1. Roufs and Blommaert then make two assumptions which permit them to interpret the slope vs delay curve as the impulse response itself, but backwards in time and delayed by an unknown amount.

The two assumptions made by Roufs and Blommaert are:

(1) when test and probe are brief pulses, and when the test amplitude is less than or equal to 30% that of the probe, then the extremum of the response always occurs at the same point in time, regardless of the delay between test and probe; and

(2) under the conditions noted, probability summation over time will have negligible effect on the visibility of the stimuli.

The first assumption can never be precisely true, since it requires that the derivative of the impulse response be constant at every point in time. No physically realizable response (except no response at all) satisfies this condition. For some impulse responses, however, this condition is approximately true, and the error introduced by the approximation may be small.

The more serious objection is to the second assumption. This may be most easily seen by considering the results predicted when probability summation over time is included or not included in the model. To do this we need a hypothetical impulse response. Although a method of estimating the impulse response has yet to be developed, we do know one empirical constraint upon its form. The modulus

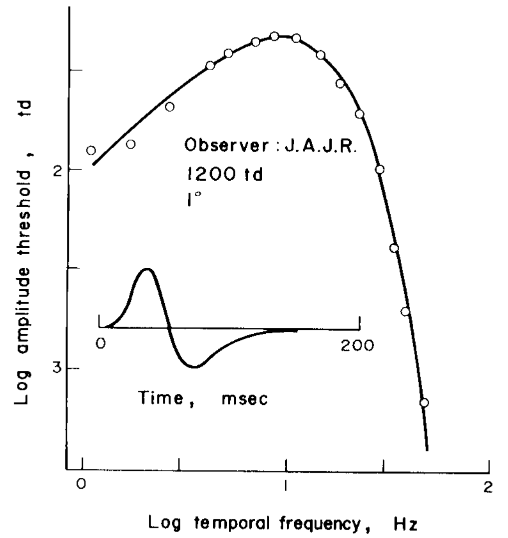

Fig. 2. The circles show sensitivities to sinusoidal modulation of the contrast of a 1° circular stimulus on a dark background, as measured by Roufs and Blommaert (1981). The smooth curve through the points is the amplitude response corresponding to the hypothetical impulse response shown in the inset.

of the Fourier transform of the impulse response is the *amplitude response*, which specifies the amplitude with which sinusoids of different frequencies pass through the system. Thus the amplitude response corresponding to a hypothetical impulse response should resemble the sensitivity of the observer to sinusoids of various temporal frequencies. Examples of this approach may be found in Kelly (1971), Roufs (1972) and Watson (1981).

A further example of this technique is shown in Fig. 2. The points are amplitude sensitivity measurements made by Roufs and Blommaert (1981) for their 1° disk target at an adapting illuminance of 1200 td. The smooth curve through the points is the amplitude response corresponding to the impulse response shown in the inset to Fig. 2. This impulse response is given by

$$h(t) = u(t)[(t/4.94)^8 e^{-t/4.94} - (1/12)(t/6.58)^9 e^{-t/6.58}] \quad (1)$$

where t is in msec and where $u(t)$ is the unit step function. This impulse response is at least plausible under Roufs and Blommaert's conditions, since it generates an appropriate amplitude response. This impulse response is similar to those derived by Kelly and Roufs from data collected under similar conditions.

To predict the effects of probability summation over time we may make use of a simple model based upon observations by Quick (Quick, 1974; Watson, 1979). In this model a temporal stimulus $f(t)$ will be at threshold when

$$1 = \int_{-\infty}^{\infty} |f(t)*h(t)|^{\beta}\, dt \quad (2)$$

where * indicates convolution and β is a parameter which usually reflects the slope of the psychometric function. A useful feature of this formula is that it can accommodate both the case of probability summation and the case of no probability summation. For no probability summation, the parameter β is set to a large value (e.g. 100). To include probability summation, β is equated to the slope parameter of the psychometric function. Watson (1979) published distributions of estimates of β for two observers in yes/no experiments. Of 104 estimates, 89 were between 3 and 7. Roufs (1974) tabulated estimates from the data of seven different investigators (whose psychophysical methods were not stated). These values of β ranged from 2.7 to 4.6, with a mean of 3.5. In the paper under discussion, an average β of 6.6 is reported ("Crozier quotient" of 0.175), but this is atypically high. These estimates appear to be derived from very small numbers of trials (an average of 33 trials/psychometric function), and are likely to be biased upwards (Nachmias, 1981). Finally, it should be noted that estimates of β covary with the false alarm rate (Nachmias, 1981), so that large individual differences may be expected in yes/no experiments.

Quick (1974) has noted that a model of this sort may reflect nonlinear integration, rather than probability summation of the filter output. For the purpose of this argument, either possibility may be entertained. Whatever its basis, this model has been found to give a good account of temporal summation (Watson, 1979).

The slope vs delay curves predicted by equations 1 and 2 for several values of β are shown in Fig. 3. Contrary to assumption 2, including probability summation makes a large difference. Without probability summation ($\beta = 100$), the predicted curve is two phased, and closely approximates the assumed impulse response. With probability summation (β between 2

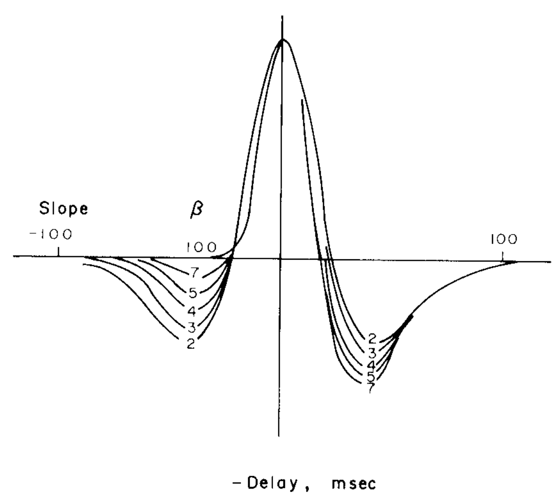

Fig. 3. Slope vs delay curves predicted by the impulse response of Fig. 2. Predictions were made with various values of β. No probability summation is represented by $\beta = 100$, probability summation by values between 2 and 7.

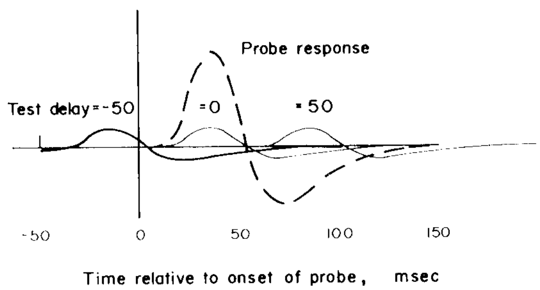

Fig. 4. An explanation of the effect of probability summation on the slope vs delay curve. Responses to the probe and to tests at three different delays are shown. The total response at each delay would be the sum of the probe response and the relevant test response. Without probability summation, all three mixtures will be detected at approximately the peak of the probe response, hence the test can have an effect only by altering the value at this peak. At a delay of −50 msec, it will reduce this peak, at 0 msec it will raise the peak, and at +50 msec it will have no effect. The resulting slope vs delay curve (Fig. 3, $\beta = 100$) reflects this outcome. With probability summation, every point in the response has an effect, since each has some probability of exceeding threshold. As a result, the test delayed by +50 msec reduces the visibility of the mixture, since its positive phase subtracts from the negative phase of the probe response. Thus when probability summation is in action, test stimuli have an effect both when they follow and when they precede the probe. This outcome is also illustrated in Fig. 3 ($\beta < 100$).

and 7), the slope vs delay curve is triphasic, and does not equal the impulse response. In other words, when probability summation does take place, a conventional biphasic impulse response like that in Fig. 2 can generate a slope vs delay function like those found by Roufs and Blommaert. Figure 4 provides a qualitative explanation of the discrepancy between predictions made with and without probability summation.

The agreement between these predictions and the data of Roufs and Blommaert could be improved by more deliberate selection of the impulse response. But since both data and predictions are subject to considerable uncertainty, it seems wiser to be content with a qualitative point: when probability summation is included, the slope vs delay curve does not equal the impulse response, and in particular, a biphasic impulse response may give rise to triphasic slope vs delay data. Since there is ample evidence in favor of probability summation (or nonlinear integration) over time (Roufs, 1974; Watson, 1979), and no evidence against it, there seems little justification for interpreting data like those in Fig. 1 as the impulse response. The data are at least as consistent with an impulse response like that shown in Fig. 2.

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