RESEARCH NOTE

SUMMATION OF ASYNCHRONOUS GRATINGS

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INTRODUCTION

The sum of two just detectable sinusoidal gratings is only slightly more detectable than either one alone, provided that their frequencies are at least an octave apart (Sachs et al., 1971). A widely accepted interpretation of this result is that the two gratings are detected by two stochastically independent channels, each of which is sensitive to only one of the two frequencies. On this theory, the compound grating is slightly more detectable than either of its constituents because of probability summation between the two channels. In terms of contrast, probability summation predicts a difference of about 1 dB between thresholds for simple and compound gratings. If the two gratings excite a single channel, then their responses should add and a difference of about 3.5 dB would result, depending somewhat upon the phases and frequencies (Graham et al., 1978). Experiments typically show about 1.2 dB of summation, consistent with probability summation among spatial frequency selective channels (Graham and Nachmias, 1971; Graham et al., 1978).

There are at least three other interpretations of this result.

(1) The gratings fail to summate because they are detected at different locations in the visual field (Limb and Rubinstein, 1977). This theory, which invokes the well-established variations in spatial sensitivity across the retina, is inconsistent with the recent demonstration that gratings confined to a small region of the periphery also fail to summate (Graham et al., 1978).

(2) There is abundant evidence that high and low spatial frequencies produce responses of substantially different time course (Robson, 1966; Tolhurst, 1975; Breitmeyer, 1975; Watson and Nachmias, 1977). Hence the responses to the two gratings may not add because they fail to superimpose in time. Presumably, the gratings would summate more effectively if displaced in time relative to one another so that their responses did coincide.

(3) Both possibilities (1) and (2) might be true. In this case the gratings would summate only if both localized in space and offset appropriately in time.

In this note we show that temporal asynchrony between the components of a compound grating does not affect its detectability. A similar result has been reported by Kasday (1978). We further demonstrate that this result is obtained whether or not the gratings are spatially localized.

METHODS

In all 3 experiments patterns were displayed on a cathode ray tube and were viewed with both eyes and natural pupils. The equipment and methods of stimulus generation were conventional, and are described for Experiments 1, 2 and 3 in Watson and Nachmias (1977), Watson (1979) and Watson and Robson (1979), respectively.

EXPERIMENT 1

In the first experiment we determined the probability of detecting gratings of 3.5 and 10.5 c/deg, both separately and in combination. Each trial was marked by a tone, and consisted of two time slots, each 25 msec in duration, separated by some asynchrony. Each grating could be presented in any trial with a probability of 0.5, and when presented, occurred in either (but not both) time slot with equal probability. Thus a given trial might contain a simple grating, a synchronous compound grating, a compound grating of some asynchrony, positive or negative, or a blank. In each of 9 sessions of 600 trials asynchronies of 53, 78, 105, and 131 msec were used. The gratings were presented in cosine phase. On each trial, the observer (ABW) reported whether he saw the stimulus.

The triangles and circles in Fig. 1 show respectively detection rates for simple and compound gratings, averaged across all sessions. The squares indicate false alarm rates. The broken line represents the detection rate for compound gratings predicted by probability summation. The line, which is essentially flat, provides a good account of the obtained detection rates; in only one case out of nine does the line fall outside the 95% confidence intervals, and the other small departures show no obvious trend.
**EXPERIMENT 2**

In this experiment 2AFC psychometric functions were collected concurrently for simple gratings of 3 and 9 c/deg and for their combination at some asynchrony. The gratings were again presented in cosine phase. In each session of 720 trials a single temporal asynchrony was used, and the three types of stimuli were presented equally often at each of 4 contrasts spanning in 2 dB steps a previously determined threshold. The duration of each component grating was 15 msec. One naive but practiced observer (LS) was used.

Each of the 17 sessions of this experiment provided three psychometric functions. The 51 sets of data were individually fit by the familiar function

\[ p = 1 - 0.5 \exp[-(c/\alpha)^\beta] \]

where \( p \) is the proportion correct, \( c \) the contrast, \( \beta \) a parameter controlling the slope of the function, and \( \alpha \) the threshold contrast, that is, the contrast at which \( p = 0.82 \). The fitting procedure provides maximum likelihood estimates of \( \alpha \) and \( \beta \), and is described by Watson (1979).

At each asynchrony, an estimate of threshold for each simple grating has been obtained by averaging (in dB) the estimates from individual sessions. The threshold for a compound from one session may then be expressed as two numbers indicating for each component the fraction of its threshold present in the compound. These two numbers may then be converted, by means of the convenient summation rule described in the appendix, into a single number which approximates the amount (in dB) by which the threshold for either component is reduced when that component is part of a compound. These threshold reductions, averaged at each asynchrony, are plotted in Fig. 2. The vertical bars enclose 4 SE, based on a standard deviation estimated from all 17 threshold reductions.

The estimates of \( \beta \) from all 51 psychometric functions have also been averaged, yielding a mean of 3.71 and SD of 0.86. By assuming equation (1) and this estimate of \( \beta \), and by supposing that incorrect responses represent guesses, we can calculate the threshold reductions expected if both components are detected by the same channel, or by different channels (Graham et al., 1978; Quick et al., 1978). These two predictions are shown by horizontal broken and continuous lines respectively in Fig. 2. Allowing \( \beta \) to vary over

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**Fig. 1.** Detection rates as a function of asynchrony for component gratings of 3.5 c/deg (open triangles), 10.5 c/deg (solid triangles), for compound gratings (circles) and for blanks (squares). The broken line is the rate predicted for compound gratings by probability summation.

**Fig. 2.** Threshold reductions for compound gratings as a function of the asynchrony between the component gratings of 3 and 9 c/deg. Reductions predicted by one and two channel models are shown.
\( \pm 2 \text{SE} \) of its mean alters these predictions by only about 0.1 dB. Note that the one channel prediction specifies the threshold reduction expected if the responses to the two gratings were superimposed in time. The data are well described by the two-channel prediction, and no asynchrony gives a threshold reduction approaching that predicted by a single channel.

Several more stringent tests have been applied to these data in an attempt to find any systematic departures from probability summation. None were successful, so we shall describe only the simplest. Within each session three contrast levels were common to both simple and compound gratings. At each of these levels, probability summation predicts that the three proportions correct should be related by

\[
p_{x+0} = 1 - 0.5(1 - p_0)(1 - p_0)
\]

A likelihood ratio test applied to each of the 17 sessions never rejects \( (p < 0.005) \) this hypothesis.

**EXPERIMENT 3**

In order to lessen the possible influence of retinal inhomogeneity we conducted a third experiment in which the gratings were windowed by multiplication with a radially symmetric Gaussian function which fell to 0.37 at a distance of 0.75° from its maximum. In addition, these patches of gratings were presented 7° above fixation in a region shown to be relatively uniform in spatial sensitivity (Robson and Graham, 1978). To make the test more bold, we used frequencies of 2 and 4 c/deg, only one octave apart. The gratings were presented in sine phase relative to the center of the spatial window. This phase can be shown by calculation to give the largest difference between single and multiple channel predictions (about 2 dB).

In order to remove possible complications introduced by temporal transients, the presentation of each component grating followed a Gaussian time course, falling to 0.37 of its maximum in 40 msec. The asynchrony of the components is specified by the time intervening between the maxima of their time courses.

Within a session, the two simple gratings and their asynchronous compound were each presented 60 times in random order. The ratio of contrasts of the two components of the compound was fixed within a session at the current estimate of the ratio of their thresholds, and the contrast of both simple and compound gratings was controlled from trial to trial by a 2AFC QUEST staircase procedure. This efficient procedure presents each trial at the current maximum likelihood estimate of threshold, and will be described in more detail in a future publication (Watson and Pelli, 1979). The observer in this experiment was ABW.

The data of each session were treated identically to those of Experiment 2, except that in the fitting procedure the value of the exponent \( \beta \) was set at 3.5 (since 60 trials do not allow a precise estimate of this parameter). This value of \( \beta \), which does not differ significantly from estimates derived from greater numbers of trials under the present experimental conditions, has also been used in the predictions. Figure 3 shows the threshold reductions at each asynchrony averaged over a minimum of 4 sessions. The vertical bars enclose \( 4 \text{SE} \). Again, the results do not depart to any appreciable extent from the two channel model, and no asynchrony gives an amount of summation consistent with the single channel model.

**DISCUSSION**

The threshold contrast for a compound grating whose components differ by at least an octave in frequency is unaffected by temporal asynchrony between the components. This is so whether or not the gratings are confined to a small region of retina of relatively uniform spatial sensitivity. These findings are entirely consistent with probability summation between independent, spatial frequency selective channels, and are inconsistent with a single channel model in which different frequencies fail to summate because they are detected at different points in space, time, or space and time.

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**APPENDIX**

Consider two stimuli of amplitudes $a$ and $b$ and thresholds $A$ and $B$. Consider a rule for combining the effects of the two stimuli which says that, at threshold,

$$ I = (a/A)^M + (b/B)^M $$

(A1)

The parameter $M$ specifies a family of summation rules, among which are addition ($M = 1$), selection of the maximum ($M = \infty$), and probability summation ($M = \beta$, where $\beta$ is the exponent of equation (1) in text).

Note that when the stimuli are added in amplitudes that are equal fractions of their thresholds, that is, when $a/A = b/B$, then at threshold for the compound,

$$ a/A = 2^{-1/M} $$

(A2)

Hence the threshold reduction for either stimulus, that is, the decibel difference between $a$ and $A$ or $b$ and $B$ is given by $6/M$. For example, the three cases noted above given threshold reductions of 6, 0, and 6 dB. The quantity $6/M$ will be a useful measure of the amount of summation obtained between two stimuli if the value of $M$ derived from data by means of equation (A1) does not differ too much upon departures from equality of $a/A$ and $b/B$; in other words, if the true summation function does not differ too much from that given by equation (A1).