MTI RADAR FOR AIRLINE OPERATIONS

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EXECUTIVE SUMMARY

This report presents work performed under the general topic of High-Speed Research, Flight-Deck Systems. The sensor of choice is an Airborne Moving-Target-Indicator (AMTI) radar, which is a Pulse-Doppler radar equipped with AMTI signal processor. Our specific area of interest is detecting airborne obstacles, while the aircraft is in the phase of final approach for landing (or during takeoff). An adjunct area, due to be addressed under a different cover, is the detection of obstacles on the runway during the same phases of flight.

The main impediment to successful obstacle detection is the interference of terrain clutter. This problem aside, even a non-AMTI radar is capable of detecting typical airborne obstacles —moving or stationary (floating)— at ranges in the order of 20 mi. In X-band frequencies, it is the ground clutter —not receiver noise or weather attenuation— against which the target signal has to compete. Depending on the geometry, this clutter is received through either the main antenna lobe or its sidelobes. To determine which antenna lobes contribute clutter, one has to find the ground intersection (for a flat terrain, it's a circle) with the sphere of radius equal to the target range. The angles from the antenna boresight to points on this line of intersection, and the antenna beam pattern, determine which lobes receive clutter. Therefore, assuming a horizontal antenna pointing, the higher the flight altitude, and the shorter the range, the farther (angularly) the deleterious sidelobes are from boresight —meaning less clutter. This is why there is no much concern about obstacle detection during the cruise phase of the flight.

In the approach-for-landing phase of the flight, the geometry becomes of concern because of the low-altitudes involved. It becomes much more likely that the *main* lobe —not a sidelobe— will intersect the ground at the same range as a potential airborne obstacle. In such a case, the obstacle and the ground clutter are similarly weighted by the main lobe, and the ground clutter will typically overwhelm the target signal —even for very close targets (such as 1 mi). This is where the AMTI emerges as a possible solution.

Various AMTI methods have been discussed in the open literature —which is our sole source of information. Our analysis shows that much of the claims in the literature are highly optimistic —at least, as far as the AMTI methods we investigated here are concerned. In this work, we analyzed AMTI methods which are based on Azimuth/Elevation (Az/El) error corrections for the equivalent-clutter direction. This can only be successful if, for any given range-gate, the clutter direction is highly correlated over time. In our simulation, we found that the equivalent-clutter direction wanders quite erratically from pulse to pulse (a phenomenon known as Glint) —even when the Pulse-Repetition-Frequency is as high as 10,000 pulses-per-second. We found that this behavior is very similar with either a Phase- or Amplitude-Monopulse

Az/El errors measurements. On the other hand, we have found that there is a much higher "spatial" correlation, that is, among neighboring range-gates at the *same* pulse (or time). Using that behavior, we came up with an algorithm that attenuates clutter to the point that a typical obstacle can be detected in the final-approach or takeoff phases.

1 INTRODUCTION – THE AMTI RADAR

In this section we give a brief review of Moving-Target-Indicator (MTI) radar systems (which are stationary) and of AMTI which is their Airborne counterpart. There are many approaches to AMTI processing. Out of these, we concentrate here on methods that either employ Phase- or Amplitude-Monopulse processing.

Let us start with the basics of MTI. A moving target returns transmitted energy at a frequency which is higher or lower than the carrier by the Doppler frequency, f_d.

$$f_d = \pm 2v_r/\lambda \ , \tag{1}$$

where λ is the carrier wavelength. The plus sign applies to an approaching target with radial velocity v_r and the minus sign applies to a receding target. Assume we are using a pulsed radar to observe a moving target. The envelope of the video voltage, after the high-frequency received signal has been converted down to baseband, can be written as

$$E_2 = E\sin(2\pi f_d t + \phi), \tag{2}$$

where ϕ is the phase shift due to range relative to that of the reference oscillator, and E is the amplitude of the video envelope. This voltage can be thought of as a pulse train at the pulse repetition frequency (PRF), f_r, modulated by a sin function or, alternatively, as a sin function sampled by a train of pulses. This signal is typically fed into two channels where one channel includes a delay of value equal to a single interpulse period, T ($T = 1/f_r$). The delayed signal can be written as

$$E_1 = E\sin[2\pi f_d(t - 1/f_r) + \phi] \tag{3}$$

The subscripts 1 and 2 on E denote the order of arrival of the pulses. Delaying E_1 by T shifts its whole train of pulses so that they coincide in time with the un-shifted train of pulses E_2. A simple canceler for stationary targets (for which $f_d = 0$) is formed by subtracting the delayed from the un-delayed signal on a pulse-by-pulse basis, that is,

$$E_r = E_2 - E_1 = 2E\sin\left(\frac{\pi f_d}{f_r}\right)\cos\left[2\pi f_d(t - 0.5/f_r) + \phi\right]. \tag{4}$$

The difference voltage is a train of pulses modulated by a cosine waveform at the Doppler frequency whose amplitude depends on the ratio between that frequency and the PRF. It is seen that there exist Doppler frequencies, in addition to $f_d = 0$, for which the output is zero. These are called blind speeds which occur when

$$f_{blind} = nf_r \ ; \qquad n \text{ an integer}, \tag{5}$$

and the corresponding blind speeds are

$$v_{blind} = n f_r \frac{\lambda}{2} \; ; \qquad n \text{ an integer} \tag{6}$$

figure= yair/tex/ps/mtifig1.eps,height=3.5in

Figure 1: MTI 1-delay filter power gain (DC suppression)

The value of the Difference signal E_r depends on the phase ϕ. In practice, this dependency is eliminated by quadrature processing, and E_r is thus treated as a complex function of time, or a phasor. Detection is performed by summing up the squares of the real and imaginary channels (the traditional I^2+Q^2) which is a value that represents power. figure 1 shows the power dependency on the ratio of Doppler-to-PRF frequencies f_d/f_r. The region of interest on this graph is around zero. In other words, this is a filter which suppresses all stationary targets (that have zero f_d). The above single-delay canceler serves to explain the basic idea of DC suppression. In practice, double-delay cancelers and more complicated digital filters are used to create deeper nulls at DC and, sometimes, also at other Doppler frequencies, such as those received from jammers in military applications. In the above we gave a simplified explanation of MTI processing methods. For more information on this subject we refer the reader to references such as [**?**], [**?**], [**?**], and [**?**].

Airborne MTI (AMTI) extends ground-based MTI into the realm of airborne applications. Since the radar platform is in motion, every point on the ground moves with respect to the radar, and thus contributes a Doppler frequency proportional to its relative speed. The basic idea here is to shift all Doppler frequencies, originating from the radar-beam footprint on the ground, by their *average*. A technique called TACCAR (time-averaged-clutter coherent airborne radar) is commonly used for this purpose. It works by adaptively shifting the intermediate frequency (IF) of the receiver using a closed-loop control system that minimizes the total Doppler leakage out of the MTI filter (*e.g.*, [**?**]). This amounts to centering the Doppler spectrum on the null of the MTI filter. However, no matter how deep that null is, the spectrum is always of a finite width, and thus will always leak through the filter to some extent. This problem is referred to in the literature as Platform Motion Effect.

The next step in AMTI development was to cancel out the above leakage. The remedy came in the form of a method called Displaced Phase Center Antenna (DPCA). There are two conceptual ways of accomplishing this. One is through using two physical antennas which are displaced along the longitudinal airplane axis, and the other is through an electronic equivalent. The idea behind using two physical antennas is that the aft antenna can, in principle, transmit

and receive a pulse from the same location in space where the fore antenna transmitted and received the previous pulse. This means that the distance between the two antennas, d, should equal the distance traveled by the aircraft during the interpulse time, that is, $d = vT$, where v is the aircraft speed. T can be controlled by an on-line adaptation of the PRF. Thus, ideally, two pulses transmitted and received from the same location in space emulate a stationary radar to which standard MTI methods can be applied. However, the problem with this method is that, in addition to the antenna spacing constraint, there are many other conditions that have to be met in order to achieve good performance in practice. For example, the two antennas have to be identical, and the aircraft has to fly with zero side slip and angle of attack. The electronic DPCA offers another alternative which is more practical to mechanize; this is explained in the following.

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Figure 2: The basic AMTI geometry

In Fig 2 we show the basic geometry. Vectors are denoted by capital letters and their length by the corresponding lower-case letters. Also, we write the three components of vectors in square brackets separated by commas, and location of points by regular brackets separated by commas. V is the velocity vector in the direction of the Y axis. The aircraft is located above the origin at point a: $(0, 0, h)$. Point c at $(x_0, y_0, 0)$ on the ground is where the antenna boresight pierces the X, Y plane. The vector pointing to this point from the aircraft is R_0. The antenna boresight is at an angle α from the velocity vector; it is found in the slanted plane, denoted by P, which is parallel to the Y axis.

We assume that the antenna pointing is performed by first lowering its axis by the depression angle γ, and then scanning in azimuth by the angle ϕ. It is seen in Fig 2 that, for a fixed γ, the antenna axis is confined during azimuth scanning to the plane denoted by Q which contains the vector R_0 and is parallel to the X axis.

The Platform Motion Effect is caused by the difference in radial velocities between that of the boresight piercing point, c, and those of all other ground points found in the antenna footprint such as point d: $(x, y, 0)$ in figure 2. For this point, the radial velocity difference is given by

$$v_e = v[\cos(\alpha + \Delta\alpha) - \cos\alpha] = -v_x \sin\Delta\alpha - 2v_y \sin^2\frac{\Delta\alpha}{2}, \tag{7}$$

where

$$v_x \triangleq v\sin\alpha; \qquad v_y \triangleq v\cos\alpha \tag{8}$$

and $\Delta\alpha$ is the difference between the α of point d and that of point c (notice they are not in the same plane). Since $\Delta\alpha$ is a small angle in the order of the beam-width, we can neglect the

second term compared to the first, so

$$v_e \approx -v_x \sin \Delta\alpha \tag{9}$$

Since we assumed that the AMTI already compensates for the radial Doppler velocity of point c, the Doppler frequency from $(x, y, 0)$, is

$$f_d = -\frac{2v_x}{\lambda} \sin \Delta\alpha \ ; \tag{10}$$

it is negative because the point d is receding compared to c, but we will ignore the sign for simplicity.

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Figure 3: The DPCA phasors compensation

In electronic DPCA the idea is to add appropriate vectors to the phasors of two consecutive pulses (from the same antenna) so as to bring them to equality, which will enable perfect cancelation in an MTI delay-line-type canceler as explained above. This is shown in Figure **??**. The phasor of the first pulse is E_1 and that of the second pulse is E_2. The angle between them, denoted by 2η, is given by

$$2\eta = 2\pi f_d T = \frac{4\pi v_x T \sin \Delta\alpha}{\lambda} \tag{11}$$

Idealized correction phasors E_{c1} and E_{c2} are added to the original received phasors at 90^0 phase angles as shown in the figure. For perfect cancelation, the length of these phasors should be

$$E_c = E_1 \tan \eta = \Sigma(\Delta\alpha) \tan \frac{2\pi v_x T \sin \Delta\alpha}{\lambda} \ , \tag{12}$$

where $\Sigma(\Delta\alpha)$ is the Sum pattern of a two- or four-lobe antenna —similar to that used in a Phase- or Amplitude-Monopulse radar. In such a radar two (or more) receivers are used; their sum provides the "Sum signal" and their difference the "Difference signal", $\Delta(\Delta\alpha)$. A uniformly illuminated phase-monopulse array [**?**], which is receiving signals from a *single* scatterer, produces the Difference signal in quadrature with the Sum signal:

$$\Delta(\Delta\phi) = j\Sigma(\Delta\phi) \tan \frac{\pi W \sin \Delta\phi}{\lambda}, \tag{13}$$

where $j \triangleq \sqrt{-1}$, and W is the distance between the phase centers of the two antenna halves (we will return to this formula later). Notice that the last equation is written in terms of $\Delta\phi$, which is the angle increment to ϕ in the Q plane associated with $\Delta\alpha$. The angle ϕ is used here because the line connecting the two phase centers is found in the Q plane. The angles $\Delta\alpha$ and $\Delta\phi$ are small —in the order of 5^0, which is the typical antenna beam-width. Therefore, we can approximate

their sin by the angles themselves, and express one in terms of the other by using a multiplying constant, C, *i.e.*, $\Delta\phi = C\Delta\alpha$. From the geometry, $\cos\alpha = \cos\gamma\cos\phi$. Differentiating that, gives $\sin\alpha\,\Delta\alpha = \cos\gamma\sin\phi\,\Delta\phi + \sin\gamma\cos\phi\,\Delta\gamma$. For a fixed altitude and range (same range-gate), $\Delta\gamma \approx 0$. Therefore $\sin\alpha\,\Delta\alpha \approx \cos\gamma\sin\phi\,\Delta\phi$, and $C \triangleq \Delta\phi/\Delta\alpha \approx \sin\alpha/(\sin\phi\,\cos\gamma)$.

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Figure 4: The electronic DPCA block diagram

It is thus seen that choosing $W = 2v_xT/C$ can provide the 90^0 phase-shifted signal to be added to the current (second) pulse and subtracted from the earlier (first) one, *i.e.*, E_{c2} and E_{c1} correspondingly. This relationship can be maintained by controlling the PRF, as was also required for physical DPCA. We still have here displaced phase centers which are needed for the realization of the Phase-Monopulse radar, but all the other requirements of the physical DPCA are dropped. The block diagram of the electronic DPCA is shown in Fig **??**. Each channel in the figure is complex, *i.e.*, it's a phasor sampled at the PRF. The T block delays the E_1 pulses so that they align with those of E_2, and thus can be subtracted on a pulse-by-pulse basis. After the final subtraction, the square-law detector performs the I^2+Q^2 operation (in radar terminology: sum-squared of the real and imaginary components of a complex signal), and the low-pass-filtering (for the PRF). That results in the power, which, at this point, carries no Doppler information. This power is ideally zero —but only for a single scatterer. In the case of many scatterers, there is another phenomenon that tends to render this kind of processing ineffective, as we discuss next.

2 PHASE-MONOPULSE RADAR

In this section we elaborate on the performance of a generic AMTI Phase-Monopulse radar in some more detail. We develop the basic theory of operation and describe our simulation.

2.1 The Peculiarities of Clutter —Stationary Case

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Figure 5: Phase-Monopulse Geometry.

Figure **??** shows the geometry of a Phase-Monopulse antenna. The two phase centers are separated by distance W, and the return signal from a single transmitted pulse arrives from

direction θ with respect to the boresight (θ used here instead of $\Delta\alpha$ or $\Delta\phi$ to simplify notation). The same signal travels an additional distance of $W \sin\theta$ to arrive at phase center #2 compared to phase center #1. This distance difference in the receiving path translates into a phase delay of

$$\psi = \frac{2\pi W \sin\theta}{\lambda} \approx \frac{2\pi W\theta}{\lambda} \tag{14}$$

Denoting the signals at the two phase centers by z_1 and z_2, $z_2 = z_1 \exp\{-j\psi\}$. Defining the Sum and Difference signals as $\Sigma \stackrel{\Delta}{=} z_1 + z_2$ and $\Delta \stackrel{\Delta}{=} z_1 - z_2$,

$$\frac{\Delta}{\Sigma} = \frac{1-\exp\{-j\psi\}}{1+\exp\{-j\psi\}} = j\tan(\psi/2) = j\tan(\frac{\pi W\theta}{\lambda}) \tag{15}$$

We see that the Difference signal is in quadrature (90^0 ahead) with the Sum signal. Also, the above equation implicitly imposes a bound on W, because, if W is too large, θ becomes ambiguous. It makes sense to require no ambiguities inside the main beam which translates to

$$\psi \leq \pi \; ; \quad \frac{2\pi W\theta}{\lambda} \leq \pi \; ; \quad W \leq \lambda/B \; , \tag{16}$$

where we equated θ with half the null-to-null power beam-width, B, of a single antenna lobe.

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Figure 6: Signal Vectors in Phase-Monopulse Radar for single (left) and two (right) scatterers.

Figure **??** shows (left) the vectors representing the complex signals z_1 and z_2 for a single radar pulse as discussed above. In the case of a single scatterer, $z_1 = a_1$ and $z_2 = a_2$, where a_1 and a_2 are the signals arriving from that scatterer at the two phase centers. In this simple case it is easy to see that the Sum and Difference signals are in quadrature due to the length equality of z_1 and z_2. However, in the case of two scatterers, shown on the right of figure **??**, $z_1 = a_1 + b_1$ and $z_2 = a_2 + b_2$, where b_1 and b_2 are the signals arriving from some other scatterer at the two phase centers. Assuming that the two scatterers are seen at *different* squint angles, the phase angles ψ and ϕ are —according to (**??**)— different. The angle α between a_1 and b_1 represents the phase difference between the two scatterers' signals as they arrive at the left phase center of the antenna. This angle, as well as the scatterers' amplitudes, are inconsequential, and, thus, were chosen arbitrarily. It is easy to see that, because $\psi \neq \phi$, the lengths of z_1 and z_2 are *different*, and, thus, their sum and difference are no longer in quadrature as they were in the case of a single scatterer.

In the general case, ground clutter is composed of infinite number of scatterers —each one has its own random phase, amplitude, and direction of arrival. Therefore, the Sum and Difference signals are generally not in quadrature, and the processing based on the relationship of (**??**) is

incorrect. Because of that, it has been suggested [**?**] to estimate θ based on the *projection* of the Difference signal on its nominal (single-scatterer) direction. As we have seen in (**??**), the nominal Δ is 90^0 in advance of Σ. So nominally, $-j\Delta$ should be in the same direction as Σ. In reality $-j\Delta$ is found at some non-zero angle δ from Σ. According to [**?**], (**??**) should be replaced by

$$\tan\frac{\pi W\theta}{\lambda} = \frac{\mid \Delta \mid}{\mid \Sigma \mid}\cos\delta = \left(\frac{-j\Delta}{\Sigma}\right)_{Re} = \frac{\Delta_{Im}\Sigma_{Re} - \Delta_{Re}\Sigma_{Im}}{\mid \Sigma \mid^2}, \tag{17}$$

where the Re and Im subscripts correspondingly denote the real and imaginary parts of the signal.

Even this modified single-pulse (or instantaneous) processing may yield a completely wrong angle-of-arrival, θ, in most practical cases, when the clutter is composed of infinite number of scatterers. Again, using the example of only two scatterers at different squint angles, completely wrong would mean that the *equivalent* θ for the combination of the two targets is out of the angular span between them. To show that, let us first write the equations for the case of the two targets discussed above:

$$\begin{aligned} \Sigma &= z_1 + z_2 = a_1 + b_1 + a_2 + b_2 \ ; \\ \Delta &= z_1 - z_2 = a_1 + b_1 - (a_2 + b_2) \end{aligned} \tag{18}$$

Without loss of generality, we can take $a_1 = 1$. Thus $a_2 = \exp\{-j\psi\}$. The second target contributes a signal of $b_1 = b\exp\{j\alpha\}$ at the left phase center (b is its absolute value), and $b\exp\{j(\alpha-\phi)\}$ at the left phase center. Thus

$$\begin{aligned} \Sigma &= z_1 + z_2 = 1 + b\ \exp\{j\alpha\} + \exp\{-j\psi\} + b\ \exp\{j(\alpha-\phi)\}\ ; \\ \Delta &= z_1 - z_2 = 1 + b\ \exp\{j\alpha\} - (\exp\{-j\psi\} + b\ \exp\{j(\alpha-\phi)\}) \end{aligned} \tag{19}$$

Using (**??**), and defining $k \triangleq \pi W/\lambda$, we can express the phases by the corresponding squint angles to the two targets, θ_a and θ_b. Thus $\psi = 2k\theta_a$ and $\phi = 2k\theta_b$, and

$$\begin{aligned} \Sigma &= 1 + \exp\{-j2k\theta_a\} + b\ \exp\{j\alpha\}(1 + \exp\{-j2k\theta_b\}) \\ &= 2\exp\{-jk\theta_a\}\cos k\theta_a + 2b\ \exp\{j(\alpha - k\theta_b)\}\cos k\theta_b\ ; \\ \Delta &= 1 - \exp\{-j2k\theta_a\} + b\ \exp\{j\alpha\}(1 - \exp\{-j2k\theta_b\}) \\ &= 2j\exp\{-jk\theta_a\}\sin k\theta_a + 2jb\ \exp\{j(\alpha - k\theta_b)\}\sin k\theta_b \end{aligned} \tag{20}$$

So, for this case, (**??**) can be expanded to

$$\tan k\theta = \frac{0.5(\sin 2k\theta_a + b^2\sin 2k\theta_b) + b\sin k(\theta_a+\theta_b)\cos[\alpha + k(\theta_a - \theta_b)]}{\cos^2 k\theta_a + b^2\cos^2 k\theta_b + 2b\cos k\theta_a\cos k\theta_b\cos[\alpha + k(\theta_a-\theta_b)]} \tag{21}$$

There are a few special cases of interest to consider. First, when the two targets are located at the same squint angle, that is, $\theta_a = \theta_b$, (**??**) resolves to $\tan k\theta = \tan k\theta_a$, so that $\theta = \theta_a = \theta_b$, which is what we would expect. So here, the radar measures the correct angle. Notice that the value of b doesn't appear because our processing includes amplitude normalization. Another set of special cases arises when $\alpha + k(\theta_a - \theta_b) = 0$. Then

$$\tan k\theta = \frac{0.5(\sin 2k\theta_a + b^2 \sin 2k\theta_b) + b \sin k(\theta_a + \theta_b)}{\cos^2 k\theta_a + b^2 \cos^2 k\theta_b + 2b \cos k\theta_a \cos k\theta_b} = \frac{\sin k\theta_a + b \sin k\theta_b}{\cos k\theta_a + b \cos k\theta_b} \tag{22}$$

When $b = 1$, this reduces to $\tan 0.5k(\theta_a + \theta_b)$, which yields a θ that equals the simple average of θ_a and θ_b. However, with $b = -1$, we get

$$\tan k\theta = \frac{\sin k\theta_a - \sin k\theta_b}{\cos k\theta_a - \cos k\theta_b} = -\frac{1}{\tan 0.5k(\theta_a + \theta_b)} \tag{23}$$

The right-hand side of (**??**) can go to infinity whenever $\theta_a + \theta_b = 2n\pi/k$; $n = 0, \pm 1, \pm 2$, etc. For $n = 0$, we have $\theta_a = -\theta_b, \quad \alpha = 2k\theta_b$. Since we defined α to be the phase of scatterer b at the left antenna phase center, its phase at the source has to be zero. The phase α would result from the geometry alone —since $\alpha = \phi$. This represents any dipole (in terms of its angular span) centered on the boresight ($\theta = 0$). In this case, the solution to the apparent squint angle of such a dipole is that of $\tan k\theta = \infty$, which is the series of solutions $\theta = \pi(m + 1/2)/k$; $m = 0, \pm 1, \pm 2$, etc. For example, with $k = 10$, we get phantom targets at $\pm 9^0, \pm 27^0, \pm 45^0$, etc. We have thus shown that a single dipole centered on boresight shows up in the AMTI receiver as a series of fictitious targets at angles that are unrelated to that of the dipole center or that of any of its elements.

The DPCA method, described in principle in Section 1, relies on receiving at two consecutive pulses the same amplitude, and a phase difference that originates from the Doppler associated with the aircraft motion alone. As long as these requirements hold —for the *equivalent* target representing the above dipole, or any other combination of scatterers— the corresponding two pulses will get cancelled by the circuit of Figure **??** even though they appear to originate from some angle that has no relation to any physical location. However, because the aircraft moves during the interpulse time, the geometry to the elements of the dipole changes, and thus the dipole appears to have moved on the ground to some other location. This is known as the phenomenon of Glint. In other words, the changing geometry due to the aircraft motion decorrelates consecutive pulses (from clutter) very fast. We will next investigate this phenomenon.

2.2 The Glint Phenomenon —Dynamical Case

As we have seen, all the scatterers inside the beam-width and any given range-gate add up to some equivalent target for that particular range-gate. The apparent angular location and

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Figure 7: Correlation coefficients for clutter —averaged over all range-gates

amplitude of the equivalent target depend on the geometry between the radar and each one of the various scatterers. Because of the aircraft forward motion, this geometry changes during the interpulse time, so that the equivalent target for one pulse is different from that seen in the next pulse. This is one form of Glint, or target scintillation, which degrades clutter rejection based on pulse-to-pulse cancelation. We now want to estimate the severity of this problem.

Let us assume that there are two scatterers on the ground inside the same range-gate. Scatterer # 1 is located at $(x_0, y_0, 0)$ and # 2 at $(x_0 + \Delta x, y_0 + \Delta y, 0)$. The initial range to both scatterers, as measured by the first pulse, is $r_0 = \sqrt{H^2 + x_0^2 + y_0^2}$. By the time the next pulse is received, the radar proceeded by a distance $d = vT$ in the Y direction (see Figure **??**). The radar distance to Scatterer # 1 decreased by $\approx dy_0/r_0$ and to Scatterer # 2 by $\approx d(y_0 + \Delta y)/r_0$. So now, there is a range difference, δr, from the radar to these scatterers equal to $d\Delta y/r_0$. Denoting the ground distance between the two scatterers by l, $\Delta y \approx lx_0/\sqrt{x_0^2 + y_0^2} \approx lx_0/r_0$ (for low altitude, H). With that, $\delta r \approx dlx_0/r_0^2$. We now take $l = r_0 B$, so that the scatterers span the beam-width, and also replace x_0/r_0 by $\sin\phi$ (ϕ is the radar Azimuth angle). This yields $\delta r \approx dB\sin\phi$. The phase difference that δr causes between the two scatterers at the second radar pulse is obtained as

$$\mu = 360\frac{2\delta r}{\lambda} = \frac{720vTB\sin\phi}{\lambda} \quad \text{degrees}, \tag{24}$$

where the factor of 2 accounts for the two-way transmit/receive pulse travel time.

As an example, say we have two scatterers with equal (zero) phases, located, at $\theta_a = -0.019$ rad and $\theta_b = 0.02$ rad, The other parameters are $\alpha = 2k\theta_b = 0.4$ rad, $b = -1$, $k = 10$. Plugging that into (**??**) yields $\theta = -8.24^0$. This is the apparent squint angle as measured on the basis of a single pulse. If we assume $v = 150$ m/s, $T = 0.1$ ms (so $d = 1.5$ cm), $\lambda = 3$ cm, $B = 50$ mrad, $\phi = 30^0$, (**??**) yields $\mu = 9^0$. Repeating the same calculation of (**??**), but with μ added to α, results in $\theta = -0.1318^0$. So, we see that two consecutive pulses measured two completely different squint angles. The DPCA method could still perform as expected even when the apparent θ has no relationship to any physical angle —but only as long as this angle is constant over time, which, as we have shown, is not the case. In reality, however, the Glint phenomenon does not have such a dramatic effect as demonstrated here (to make a point); its effect is rather to decorrelate the angle measurements as a function of the interpulse travel distance of the radar platform.

We used our phase-Monopulse simulation (see Section 2.6) to obtain estimates of the Glint phenomenon in the general case, when a large number of scatterers is involved. Figure **??** shows

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Figure 8: Interpulse errors averaged over all range-gates

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Figure 9: Clutter correlation coefficients for 16 range-gates of 12.5 m

experimental correlation coefficients, averaged over 32 range-gates, for the Az and El errors as a function of the interpulse travel distance, vT, as measured in wave-lengths. For example, the azimuth error measurement resulting from a single pulse is only 80% correlated with that measured from the previous (or next) pulse, if the platform advanced by 1.5 wave-length between these pulses. For $\lambda = 3$ cm, this translates into 4.5 cm of interpulse travel. It can be seen that the correlation coefficient for the El errors is much closer to unity. We also show the correlation coefficient for $\cos(\alpha)$ of (**??**), because it results from the angular errors, and is the one directly related to the eventual double-pulse (or another-filter) cancelation error.

The above correlation coefficients can be used to yield estimates of the expected errors, as shown in Figure **??**. These are obtained —each from its corresponding correlation coefficient, ρ— by using the relationship

$$e = \sigma\sqrt{1-\rho^2} \tag{25}$$

For the above example, the expected Az error, between the instantaneous measurements at any two adjacent pulses, is $0.019324\sqrt{1-0.8^2} = 0.0116$, where 0.019324 is the measured Az standard deviation (same for all pulses). The expected Az and El errors are shown for information only. However, the directly relevant error is that of the phase between two adjacent pulses, because it corresponds to 2η of (**??**), and thus, roughly, represents the value of the unavoidable residue remaining after 2-pulse cancelation.

Notice that the phase error behaves like a quadratic, whereas the angular errors appear to be linear. The reason is that there are two factors in (**??**) which cause the phase error to increase with T; one is the correlation coefficient of $\Delta\alpha$ (very close to that of $\Delta\phi$), which increases linearly with T, and the other is T itself, which appears in the equation. The phase-error graph can, in principle, be used as follows. When the interpulse travel is 4.5 cm as above, we read $2\eta = 0.114$. So, the clutter suppression achievable due to using a 2-pulse canceler is 1/0.114=8.77 in amplitude, or 18.8 dB. If the platform travels only 1.5 cm, then $2\eta = 0.015$, and the corresponding numbers are 66.7 and 36.5 dB. This might suggest that increasing the PRF (decreasing T) will allow for effective clutter rejection. However, a similar effect also happens to the *target* (or obstacle) signal.

Since the obstacle signal must appear in some specific range-gate, it is relevant to ask how

figure=/home/windchime/u/yair/tex/ps/rangegates2.ps,width=4.5in

Figure 10: Clutter correlation coefficients for 16 range-gates of 2.5 m

useful the *average* correlation coefficient is. To answer this question, we ran the simulation so as to evaluate the correlation coefficient for each of 16 range-gates. That produced the results shown in Figures **??** (range-gate of 12.5 m) and **??** (range-gate of 2.5 m). These results correspond to the $\cos\alpha$ graph of Figure **??**. For clarity, the range-gate values were not specified in these figures, but we found that adjacent range-gates produced disparate graphs. We could not discover any regularity in these graphs. However, we do observe the following: 1) some graphs appear to be periodic, 2) some go to negative values, 3) the range-gate size has little effect on the results. We will return to the question of optimizing T for Target-to-Clutter maximization in Section 2.5.

2.3 Comparison With Amplitude-Monopulse Clutter Rejection

Here we show that Amplitude Monopulse receiver suffers from the same detrimental clutter properties that undermine the Phase Monopulse receiver.

Let us consider an example of two scatterers: one of unity amplitude, zero phase, and located on boresight; the other of amplitude a, phase α, and squint angle θ. The Sum signal is, thus, $1 + a\exp\{j\alpha\}$, and the Difference signal is $ak\theta\exp\{j\alpha\}$, where k is the slope, or gain, of the Azimuth error channel. In Amplitude Monopulse, like in Phase Monopulse, there might be phase differences between the Sum and Difference channels. Therefore, similar to (**??**), we project the Difference vector on the Sum vector. Since here the nominal Sum and Difference vectors are in phase, (**??**) is replaced by

$$\theta_{equiv} = \left(\frac{\Delta}{\Sigma}\right)_{Re} = \frac{\Delta_{Re}\Sigma_{Re} + \Delta_{Im}\Sigma_{Im}}{\mid \Sigma \mid^2} = a\theta\frac{a + \cos\alpha}{1 + 2a\cos\alpha + a^2} \tag{26}$$

When $a = -1$, and $\alpha = 0$ (**??**) reduces to $\theta/2$, but this involves the unstable operation of dividing zero by zero. Although mathematically, the limit of (**??**) is $\theta/2$, in practice one might get any unpredictable result. So, our conclusion is that neither Phase- nor Amplitude-Monopulse processing can overcome the intrinsic detrimental clutter properties that cause the equivalent clutter "target" to change its apparent azimuth from pulse to pulse.

2.4 Geometrical-Errors Calculation

Here we want to derive the *geometrical* relationship between the radar Az/El errors and the effective target direction —ignoring the Glint problem for the moment. Referring to Figure **??**,

the radar axis is pointing in the direction of the vector R_0, as given by the angles (γ_0, ϕ_0). The direction of the vector R, pointing to the effective target, is given by the angles (γ, ϕ). We want to calculate (γ, ϕ) based on the Az/El angular errors between the above two vectors.

Let us denote the tangents of the Az and El error angles by Δx and Δz respectively; these increments are aligned with the radar (X', Z') coordinates (the antenna pointing in the Y' direction). Unit vectors in the X' and Z' direction can be expressed in terms of the earth-system unit vectors as

$$\begin{aligned} 1_{x'} &= [\cos\phi_0,\ -\sin\phi_0\cos\gamma_0,\ \sin\phi_0\sin\gamma_0]\ , \\ 1_{z'} &= [0,\ \sin\gamma_0,\ \cos\gamma_0] \end{aligned} \tag{27}$$

The unit vectors in the R_0 and R directions can be written as

$$\begin{aligned} 1_{R0} &= [\sin\phi_0,\ \cos\phi_0\cos\gamma_0,\ -\cos\phi_0\sin\gamma_0]\ , \\ 1_R &= [\sin\phi,\ \cos\phi\cos\gamma,\ -\cos\phi\sin\gamma] \end{aligned} \tag{28}$$

The unit vector 1_R can also be expressed in terms of the Az/El errors as

$$1_R = \frac{1_{R0} + \Delta x \cdot 1_{x'} + \Delta z \cdot 1_{z'}}{\sqrt{1 + \Delta x^2 + \Delta z^2}} \tag{29}$$

Plugging the expressions for the $1_{x'}$ and $1_{z'}$ unit vectors into the last equation, results in

$$\begin{aligned} 1_R = [&\sin\phi_0 + \Delta x\cos\phi_0,\ \cos\phi_0\cos\gamma_0 - \Delta x\sin\phi_0\cos\gamma_0 + \Delta z\sin\gamma_0, \\ &-\cos\phi_0\sin\gamma_0 + \Delta x\sin\phi_0\sin\gamma_0 + \Delta z\cos\gamma_0]/\sqrt{1 + \Delta x^2 + \Delta z^2} \end{aligned} \tag{30}$$

Equating the components of 1_R in (**??**) and (**??**), we have:

$$\begin{aligned} \sin\phi &= \frac{\sin\phi_0 + \Delta x\cos\phi_0}{\sqrt{1 + \Delta x^2 + \Delta z^2}} \\ \cos\phi\cos\gamma &= \frac{\cos\phi_0\cos\gamma_0 - \Delta x\sin\phi_0\cos\gamma_0 + \Delta z\sin\gamma_0}{\sqrt{1 + \Delta x^2 + \Delta z^2}} \\ -\cos\phi\sin\gamma &= \frac{-\cos\phi_0\sin\gamma_0 + \Delta x\sin\phi_0\sin\gamma_0 + \Delta z\cos\gamma_0}{\sqrt{1 + \Delta x^2 + \Delta z^2}} \end{aligned} \tag{31}$$

which constitutes a set of three consistent equations for the two unknown angles ϕ and γ. The solution is

$$\begin{aligned} \phi &= \sin^{-1}\left[\frac{\sin\phi_0 + \Delta x\cos\phi_0}{\sqrt{1 + \Delta x^2 + \Delta z^2}}\right] \\ \gamma &= \sin^{-1}\left[\frac{\sin\gamma_0(\cos\phi_0 - \Delta x\sin\phi_0) - \Delta z\cos\gamma_0}{\sqrt{(\cos\phi_0 - \Delta x\sin\phi_0)^2 + \Delta z^2}}\right] \end{aligned} \tag{32}$$

Once we have found ϕ and γ, we can get α from

$$\cos\alpha = \cos\phi\cos\gamma \ , \tag{33}$$

and, in theory, predict the exact Doppler frequency created by the equivalent clutter target. Knowing that, we can frequency shift the received signal down to baseband and expect to get all pulses in phase —ready for pulse subtraction. We have thus developed a processing method, similar to the one described in Figure **??** and **??**, which has some advantages over that method. First, there is no need for phasors compensation, second, there is no need for the approximation we did in neglecting the v_y term of (**??**), and third, there is no need to approximate $\Delta\alpha$ by $\Delta\phi$ as was done in (**??**). However, recall that the above is a purely geometrical relationship, and can only translate measured errors —whether accurate, or corrupted by Glint— into the equivalent-target viewing angles ϕ and γ.

2.5 Clutter Cancelation

Here we elaborate on the clutter-cancelation technique that was introduced earlier in its simplest form of 2-pulse subtraction. It is more useful to use a Low-Pass-Filter of second order for that purpose because there is better control on the filter characteristics, that is, the width of the stop band and the flatness of the pass band.

figure=/home/windchime/u/yair/tex/ps/double.ps,width=3.5in

Figure 11: Low-Pass-Filter for Clutter Cancelation

figure=/home/windchime/u/yair/tex/ps/freq.ps,width=4in

Figure 12: Frequency Response of Low-Pass-Filter

Figure **??** shows a discrete second-order Low-Pass-Filter. The $1/Z$ blocks represent a delay of T in the Z-Transform domain, and the k_1, k_2 blocks are two gains to be determined. The Transfer-Function of this circuit is

$$\frac{y}{x} = \frac{(Z-1)^2}{Z^2 - (k_1+k_2)Z + k_1} \ , \tag{34}$$

and its frequency response for some combinations of k_1 and k_2 is shown in Figure **??**; it is obtained by replacing Z with $\exp\{j\omega\}$ and scanning ω in the range of 0 to 1. We chose $k1 = 0.6, k2 = 0.8$. In order to avoid an initial transient, we initialize the outputs of the two delay blocks to zero.

As we have shown above, the apparent clutter "target" —being composed of all the scatterers inside any single range-gate— appears at some effective "center of gravity"–direction per each

transmitted/received pulse. The aircraft motion during the interpulse time causes Glint, that is, instability in the apparent direction. Analyzing the Glint statistics turns out to be unwieldy and not very useful, so we opted for deriving this statistics through Monte-Carlo simulation; that will be discussed next.

In Section 2.2 we already introduced the experimental data for the cross-correlation and the resulting expected phase errors between any two consecutive measurements (pulses) due to clutter. Here we discuss the tradeoffs of the interpulse time, T, with respect to signal-to-clutter optimization.

In consecutive-pulse clutter cancelation, we have two clutter vectors, of some length C, with the clutter phase-error, say ϵ_c, between them. Similarly, we have two such vectors for an obstacle that travels at a ground speed v_t. Their lengths is denoted by S, and the phase angle between them is $\epsilon_s = 4\pi v_t T/\lambda$. The obstacle and clutter vectors at the first pulse have some random phase angle, ϕ, between them. The vector sums of clutter-plus-obstacle amplitudes at the first and second pulses can thus be written as

$$\begin{aligned} E_1 &= C + S\exp\{j\phi\} \\ E_2 &= C\exp\{j\epsilon_c\} + S\exp\{j(\phi + 4\pi v_t T/\lambda)\} \end{aligned} \tag{35}$$

After 2-pulse cancelation, we get the residual amplitude

$$\Delta E = E_2 - E_1 = C(\exp\{j\epsilon_c\} - 1) + S\exp\{j\phi\}(\exp\{j4\pi v_t T/\lambda\} - 1) \tag{36}$$

The residual power, averaged over the uniformly-distributed phase, ϕ, is

$$P_{C+S} = \overline{\Delta E \Delta E^*} = 4C^2 \sin^2 \epsilon_c/2 + 4S^2 \sin^2 2\pi v_t T/\lambda\,, \tag{37}$$

where the superscript * denotes complex conjugation. The signal-to-clutter ratio can thus be written as

$$\text{SCR} \triangleq \frac{P_{C+S}}{P_C} = 1 + \frac{S^2 \sin^2 2\pi v_t T/\lambda}{C^2 \sin^2 \epsilon_c/2} \tag{38}$$

Since the above phase angles are small, we can approximate the sin by its argument. So we get

$$\text{SCR} \approx 1 + \left(\frac{S}{C}\right)^2 \left(\frac{4\pi v_t T}{\lambda \epsilon_c}\right)^2 \tag{39}$$

Our goal is, of course, to maximize the SCR with respect to T. Let us use the results for the average (over all range-gates) clutter phase-error from Figure **??**. The phase-error graph can be represented as a quadratic, that is, $\epsilon_c \propto T + aT^2$ (with $a = 13,000$), because $\epsilon_c = 0$ for $T = 0$. Thus, from (**??**), SCR $\propto 1/(1+aT)^2$. Since a is positive, the SCR function has no extremum, and

it achieves its maximum when $T = 0$. This result is not very surprising because we have ignored all sources of electronic noise —assuming that the clutter dominates the noise. It is also not very practical, because, as we have seen earlier, the clutter correlation coefficients, as averaged over all range-gates, are not indicative of those for any given range-gate. If we repeat the same exercise for range gate # 6 (for which we later present simulation results), we get a distinct maximum at $l = 3.5\lambda$. In general, some of the range gates yield maximum SCR at about the same l, and some have no extremum and yield maximum values at $l = 0$. The simulation results presented in Section #4 will shed some more light on the subject of PRF (or T) optimization.

2.6 Scenario and Radar-Signal Generation for Phase Monopulse

In the simulation, we separate the generation of radar signals from their processing; that approach is taken to facilitate substitution of the simulated signals by real radar data.

In our scenario, the aircraft is flying level at constant altitude, H, and velocity, V, along the Y axis, as shown in Figure 2. The antenna is pointing in the R_0 direction as determined by the angles γ and ϕ. We assume a flat terrain having a radar scatterer every, say, 5 m in X and in Y. For each scatterer, we create two normally-distributed and independent random numbers, having a variance of some σ^2, to represent the real, Γ_{Re}, and imaginary, Γ_{Im}, components of the scatterer's radar cross-section. Thus, each scatterer has a Rayleigh amplitude distribution and a uniform phase distribution. Based on the assumed power beam-width , B, of the antenna, we limit the patch of terrain that contributes clutter to be $\pm 3B$ in both the X and Y coordinates. The patch is centered around the ground point c of Figure 2. That way we can include clutter contributions through the main antenna lobe as well as through the most-significant sidelobes.

For Phase-Monopulse, the antenna is assumed to have 4 feeds that create 4 identical radiating lobes shifted horizontally and vertically by the feeds' lateral separation, W, in the X' and Z' antenna coordinates (the antenna boresight is in the Y' direction). Each lobe is assumed to have a one-way amplitude pattern of sinc^2, that is,

$$G(\epsilon) = \frac{\sin^2(\pi\epsilon/B)}{(\pi\epsilon/B)^2} \ , \tag{40}$$

where $\epsilon^2 = \epsilon_x^2 + \epsilon_y^2$, and the latter are the squint angles from boresight in the antenna coordinate system.

Generating the radar clutter signal for a single pulse consists of integrating the contributions from all scatterers in the above-defined ground patch —given the instantaneous geometry. So, for each scatterer we do the following:

1. Calculate the nominal range, r, from the scatterer to the antenna center. Since the range is in the order of kilometers, whereas range increments per pulse are in the order of centimeters, the computation accuracy becomes of concern. Therefore we calculate the range to each scatterer incrementally by using the updated range derivative. Thus, the range for the nth pulse for some scatterer is given by

$$r_n = r_0 - vT\left(\frac{y_0}{r_0} + \frac{y_1}{r_1} + \cdots + \frac{y_{n-1}}{r_{n-1}}\right) \tag{41}$$

2. Transform the scatterer's earth coordinates into radar coordinates.

3. Using the x', y', z' scatterer's radar coordinates, calculate the approximate squint angles $\epsilon_x = x'/y'$ and $\epsilon_z = z'/y'$ and the total angular error ϵ.

4. Find the antenna gain, $G(\epsilon)$, in that direction by using the normalized total angular error, ϵ/B, as the argument to the sinc-squared function.

5. Multiply the real and imaginary Radar-Cross-Section (RCS) of that scatterer by the antenna gain, that is, $\sigma_{Re} = \Gamma_{Re}G(\epsilon)$, and $\sigma_{Im} = \Gamma_{Im}G(\epsilon)$.

6. Calculate the nominal phase delay due to the range between the scatterer and the antenna center. That is given by $\phi_0 = 4\pi r/\lambda$. Notice that the range is counted twice for the transmit and receive paths.

7. Calculate the incremental horizontal and vertical phase corrections due to the lateral shifts of the four antenna feeds with respect to the antenna center. That is based on the following:

$$\begin{aligned} r_\pm &= \sqrt{(x' \pm d_h)^2 + y'^2 + (z' \pm d_v)^2} \\ &= \sqrt{x'^2 + y'^2 + z'^2 + d_h^2 + d_v^2 \pm 2x'd_h \pm 2z'd_v} \approx r \pm \frac{x'd_h}{r} \pm \frac{z'd_v}{r}\ , \end{aligned} \tag{42}$$

where $r_\pm$ is the range to one of the feeds, (x',y',z') are the scatterer's coordinates in the antenna coordinate system, $d_h = d_v = W/2$ are the feed horizontal and vertical shifts from the antenna center (they are taken here to be equal, but, in general, they can be different). Because the X' axis points to the antenna right, and the Z' axis points up, the range to the right-up feed, for example, is obtained by using the minus signs on the two correction terms above. The signs for the other three feeds are determined accordingly. Notice that these corrections are required on the receive path only, because it is assumed that either all four feeds are used in unison, or an additional center feed is used for transmit.

8. Add up the phases due to range and feed shifts for the four feeds, that is,

$$\phi_{\text{left_down}} = \phi_0 + k(+x' + z'$$