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Visible Contrast Energy Metrics for Detection and Discrimination

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ABSTRACT

Contrast energy was proposed by Watson, Barlow, & Robson (Science, 1983) as a useful metric for representing luminance contrast target stimuli because it represents the detectability of the stimulus in photon noise for an ideal observer. We propose here the use of visible contrast energy metrics for detection and discrimination among static luminance patterns. The visibility is approximated with spatial frequency sensitivity weighting and eccentricity sensitivity weighting. The suggested weighting functions revise the Standard Spatial Observer (Watson & Ahumada, J. Vision, 2005) for luminance contrast detection , extend it into the near periphery, and provide compensation for duration. Under the assumption that the detection is limited only by internal noise, both detection and discrimination performance can be predicted by metrics based on the visible energy of the difference images.

Keywords: visual discrimination, visual detection, luminance contrast, contrast sensitivity function, contrast energy, internal noise, ideal observer.

INTRODUCTION

The goal of this paper is to provide methods for predicting detection and discrimination performance for static luminance contrast images based on a single channel model that we have previously developed for detection[1] and discrimination.[2] The energy version of the Standard Spatial Observer metric [1] is updated to allow for its extension into the near periphery, variations in the CSF with duration, and an explicit model that allows different performance levels to be used to define the threshold. The ideal observer discrimination model used to predict acuity [2] is related to a corresponding contrast discrimination metric and simple examples are provided to illustrate its use. Matlab programs are also provided.

Contrast Energy

Contrast energy was proposed by Watson, Barlow, & Robson [3] as a useful metric for representing luminance contrast target stimuli because it represents the detectability of the stimulus in photon noise for an ideal observer. Contrast energy is computed from the digital contrast image C(x,y) using the pixel area dx dy, and the energy equivalent duration dt,

$$E = dx\ dy\ dt\ \Sigma_{x,y}\ C(x,y)^2 \quad (1)$$

If the signal has been windowed by a temporal function $0 \le T(t) \le 1$, the energy equivalent duration dt is

$$dt = df\ \Sigma_t\ T(t)^2, \quad (2)$$

where df is the frame duration.

Contrast energy is conveniently represented on a decibel scale relative to the best performance of observer HB [3].

$$dBB = 10 \log_{10}(E / 10^{-6} \text{ deg}^2 \text{ sec}). \quad (3)$$

VISIBLE ENERGY METRIC FOR DETECTION

The inputs for the detection metric are a digital luminance image, L(x,y), the pixel area, dx dy, in deg^2, and the duration, dt, in sec.

Visible contrast image

Watson and Ahumada[1] computed the contrast image C(x,y) from a luminance image L(x,y) by subtracting and then dividing by the background luminance level L0.

$$C(x,y) = (L(x,y) - L0) / L0. \quad (4)$$

The contrast image C(x,y) is filtered by a frequency domain contrast sensitivity filter, CSF(fx,fy), and then multiplied by the space domain eccentricity function S(x,y) to obtain the visible contrast image, Cv(x,y).

$$Cv(x,y) = S(x,y)\ DFT^{-1}(DFT(C(x,y))\ CSF(fx,fy)), \quad (5)$$

where DFT is the digital Fourier transform and DFT^{-1} is its inverse.

An alternative approach is to base the contrast on a local luminance function. The luminance image is first filtered by an optical blur filter, O(x,y), to obtain the optically blurred luminance image, Lo(x,y),

$$Lo(x,y) = L(x,y) * O(x,y) = DFT^{-1}(DFT(L(x,y))\ DFT(O(x,y))) \quad (6)$$

where the * indicates convolution. Next, a background luminance image, Lb(x,y) is computed as a weighted average of the optically blurred image further blurred by a background blur function, B(x,y), and the also doubly blurred previous background, B0(x,y).

$$Lb(x,y) = a(dt)\ Lo(x,y) * B(x,y) + (1-a(dt))\ B0(x,y), \quad (7)$$

where the weight a(dt) is a function of the duration, taking values between 0 and 1.

The contrast image, C(x,y), is then computed from the optically blurred image by subtracting and dividing (point by point) the background image,

$$C(x,y) = (\ Lo(x,y) - Lb(x,y)) / Lb(x,y). \quad (8)$$

Finally, the visible contrast image, Cv(x,y), is computed by multiplying (point by point) the contrast image by an eccentricity sensitivity function, S(x,y),

$$Cv(x,y) = C(x,y)\ S(x,y)\ . \quad (9)$$

Although slightly more complex, the latter formulation allows the signal to be added to a non-uniform background as demonstrated by Bowen and Wilson.[4] It separates the neural surround component of the CSF from the usually primarily optical center component and emphasizes that the zeroing function of the background may be separated from the gain-setting function. Since both the background during the signal and the preceding background contribute to the local luminance, the dependence of the relative weighting of the center and surround is naturally associated with the signal duration.

Visible Contrast Energy

The visible contrast energy metric for detection is then computed using the standard contrast energy formula[3],

$$Ev = dx\ dy\ dt\ \Sigma_{x,y}\ Cv(x,y)^2. \quad (10)$$

This is conveniently represented on a decibel scale relative to the best performance of observer HB.[3]

$$dBV = 10 \log_{10}(Ev / 10^{-6} \deg^2 \sec). \quad (11)$$

Convenient Functions and Parameters

When the visible contrast image is computed using Equation 5, the contrast sensitivity function may be computed as

$$CSF(fx,fy)) = \exp(-f / 12\ cpd) - a(dt) \exp(-(f / 2\ cpd)^2), \quad (12)$$

where f is the radial spatial frequency, $f=\sqrt{(fx^2+fy^2)}$.

When the local luminance approach is used, corresponding functions are

$$O(fx,fy) = \exp(-f / 12\ cpd)\ , \quad (13)$$

$$DTF(B(x,y)) = \exp(-f / 2\ cpd) \quad (14)$$

The suggested background weight function is

$$a(dt) = 1 - \exp(-dt/ 0.13 \sec). \quad (15)$$

Finally, a suggested sensitivity function is,

$$S(x,y) = 1/(1+4.4(1- \exp(-r / 6.42 \deg))), \quad (16)$$

where $r = \sqrt{(x^2+y^2)}$.

Appendix A contains Matlab code implementing the above calculations.

Parameter estimation

The sensitivity function (Equation 16) is the inverse of a function that has been fit to human cone spacing data.[5] If each cone contributes a constant amount of independent noise to a pixel, the noise standard deviation will be proportional to the square root of the density, and the signal will be proportional to the density, so the resulting sensitivity (signal-to-noise ratio) will be proportional to the square root of the density, i. e. the inverse of the cone spacing. We fit the spacing function in degrees for eccentricities out to 20 deg by

$$s(r) = 0.592 + 2.605\,(1 - \exp(-r / 6.424\ \text{deg}). \quad (17)$$

The square root of the density normalized to be 1 when r = 0, will be

$$S(x,y) = S(r) = 1/s(r) = 1/(1+ 4.4\,(1-\exp(-r /6.42\ \text{deg}))). \quad (18)$$

The 41 Modelfest[6] mean thresholds (omitting the noise and San Francisco images) were then used to find the CSF parameters, giving a center frequency cutoff of 7.31 cpd, a surround cutoff of 1.889 cpd, and a surround weight of 0.849. The equivalent energy duration in the Modelfest experiment was 0.23 sec, so the corresponding surround weight time constant is 0.122 sec. When the local luminance approach was used, the estimated parameters were optical cutoff , 7.33 cpd; background blur cutoff, 2.14 cpd; and time constant 0.141 sec.

The recommended time constant is just the average of the two estimates.

If the filters were Gaussian, the simple CSF surround cutoff fs should be the combination of the optical filter cutoff fo and the background filter fb,

$$fs = 1/(1/fo^{2} + 1/fb^{2})^{0.5} = 1/(1/7.31^{2} + 1/2.14^{2})^{0.5} = 2.0538. \quad (19)$$

The suggested value, fs = 2, looks like an average of the two estimates, but it really is just a rounding off of the background cutoff fb. When the optical cutoff is increased to 12 cpd, its effect on the background cutoff becomes negligible.

Watson and Ahumada [2] found that the Modelfest-derived center frequency cutoff was much too low to model acuity. Deely and Drasdo [7] have modeled the optical transfer function as an exponential to a power, where both the cutoff and the power are a function of the pupil size P in mm.

$$O(f) = \exp(-f/(20.9-2.1\,P)^{1.3 - 0.07\,P} \quad (20)$$

The pupil size giving an exponent of 1 is 4.3 mm and the corresponding cutoff is 11.9 cpd, which rounds to 12.

A METRIC-VALIDATING MODEL

A simple model for detection is to assume that internal white noise is added to the visible contrast image and that detection or recognition is then performed by an ideal observer. If the individual pixels of the noise are independently normally distributed with mean zero and standard deviation σ, it is convenient to represent the noise by its expected contrast energy per pixel,

$$N = dx\,dy\,dt\,\sigma^{2}. \quad (21)$$

N is usually referred to as the two sided noise spectral density.

Yes-No Performance

Signal detection theory [8] shows that the ideal observer in a Yes-No detection task in white noise cross multiplies the noisy possible signal with the potential signal Cv(x,y), forming a normalized decision variable Z which has unit variance and whose mean

$$E(Z) = d' = \sqrt{(\Sigma x,y(Cv(x,y)^{2})/s^{2})} = \sqrt{(Ev/N)}, \text{ when the signal is present (Signal+Noise)},$$
$$= 0, \text{ when the signal is absent (Noise Alone)}. \quad (22)$$

For a criterion c, such that Pr(Yes) = Pr(Z>c), the hit and false alarm rates are given by

$$Pr(Y|Signal+Noise) = Fz(d' - c)\ ;\ Pr(Y|\ Noise\ Alone) = Fz(-c), \quad (23)$$

where Fz is the cumulative standard normal distribution. The value of d' can be computed from the hit and false alarm rates using the inverse of Fz, Fz^{-1} as

$$d' = Fz^{-1}(Pr(Y|Signal+Noise)) - Fz^{-1}(Pr(Y|Noise\ Alone)). \quad (24)$$

In the Yes-No case, when d' = 1, Ev = N.

Two-Interval Forced Choice Performance

In the two-interval forced-choice (2IFC) procedure with a signal in either the first or second interval with equal likelihood, the ideal observer cross multiplies as above to obtain a normalized Z1 for the first interval and a value Z2 for the second, takes the difference Z1 – Z2, and responds interval 1 if the variance normalized variable $Z = (Z1 - Z2) / \sqrt{2}$ is greater than zero.

We can define the 2IFC detectability index d' as

$$d' = E(Z \mid \text{interval } 1) - E(Z \mid \text{interval } 2)$$
$$= \sqrt{(E/N)}/\sqrt{2} - (-\sqrt{(E/N)}/\sqrt{2}$$
$$= \sqrt{(2Ev/N)}. \quad (25)$$

The 2IFC can be regarded as a Yes-No experiment where the signal alternatives are signal in interval 1 or 2 (S1 or S2) and the response bias has been set so that $Pr(R1|S1) = 1 - Pr(R2|S1)$. Using Equation(20) we see that in the 2IFC case

$$d' = Fz^{-1}(Pr(R1|S1)) - Fz^{-1}(Pr(R1|R2)) = Fz^{-1}(Pr(R1|R1)) - Fz^{-1}(1-Pr(R1|R1)) = 2\, Fz^{-1}(Pc) \quad (26)$$

and

$$Pc = Fz(d' / 2). \quad (27)$$

For the Modelfest4, the threshold was defined to be $Pc = 0.84 = Fz(1)$, corresponding to $d' = 2$. Thus at this threshold, $2\, Ev/N = 4$, and

$$Ev = 2\, N. \quad (28)$$

Estimating N

Watson and Ahumada[1] estimated a contrast gain factor G to predict the mean Modelfest thresholds such that

$$G^2\, dx\, dy\, \Sigma_{x,y}\, Cv(x,y)^2 = G^2\, Ev/dt = 1. \quad (29)$$

Substituting Equation 28 for Ev gives

$$N = dt / (2\, G^2). \quad (30)$$

For the functions and parameters above, the sensitivity estimates are G = 452 and 453 for the direct CSF and the local luminance methods, respectively. The resulting estimate of N is –2.5 dBB (re 10–6 deg2sec).

Equations 22 and 24 for the Yes-No case and Equations 25 and 27 for the 2IFC case give predictions for detection based only on the visible contrast energy metric Ev and the internal noise level parameter N.

DISCRIMINATION

To simplify the notation for discriminating among M equally likely visible contrast images, we will drop the space indices x and y and regard the visible contrast images as vectors,

$$Cv(x,y,j) = Cv(j),\ j=1, M. \quad (31)$$

In the presence of a white noise image vector W with power spectral density N, the unbiased ideal observer picks the image k with the smallest distance to the noisy image. If image j is presented, the squared distance to the kth image is

$$\| Cv(j) + W - Cv(k) \|^2 = \|Cv(j) + W\|^2 + \|Cv(k)\|^2 - 2\, (Cv(j)\bullet Cv(k) + W\bullet Cv(k)), \quad (32)$$

where the • indicates the inner product of images regarded as vectors and $\|X\|^2 = X\bullet X$. The first of the three terms on the right in Equation 27 is not a function of k, so the observer is trying to find the k that maximizes

$$Cv(j)\bullet Cv(k) - 0.5\, \|Cv(k)\|^2 + W\bullet Cv(k) \quad (33)$$

Simulation of this model seems to involve many calculations. For a high resolution image, just the calculation of a simulated W involves many operations. However, as we have pointed out,[2] the first two terms on the right side of Equation 32 only involve components of the M by M matrix

$$S = (S_{j,k}) = (Cv(j)\bullet Cv(k)). \quad (34)$$

The M random variables W•Cv(k) are normally distributed with mean zero and covariance matrix S, so they can be simulated by multiplying a sample of M independent Gaussians by F, a factorization of S such that

$$S = F\, F^T. \quad (35)$$

Since S is a covariance matrix the singular value decomposition of S provides an M by M orthonormal matrix U and a diagonal matrix D such that $U\, D\, U^T = S$.

$$F = U\, D^{0.5} \quad (36)$$

gives an appropriate factorization.

When all the Cv(k) have the same energy, the ideal observer becomes a cross-correlator, looking for the k that maximizes

$$Cv(j) \bullet Cv(k) + W \bullet Cv(k). \quad (37)$$

Orthogonal, equal energy visible contrast images

If the M images are orthogonal, $Cv(j) \bullet Cv(k) = 0$, and equal in length, $L = \|Cv(j)\|$, and, hence, energy, all of the cross products are equal to zero except for the correct one which will be L^2. The M noises are all independent with variance $\sigma^2 L^2$. Dividing these noisy cross products by σ L, the ideal observer will select the largest of M variables where one of them has a mean of L/σ and a variance of 1 and the other M–1 have a mean of 0 and a variance of 1. If we define

$$\delta = L/\sigma = \sqrt{Ev / N}, \quad (38)$$

then the probability of a correct discrimination Pc is

$$Pc = \int Fz(x)^{M-1} fz(x-\delta)\, dx, \quad (39)$$

where Fz and fz are the cumulative and density distribution functions of the standard normal. Figure 1 shows the resulting performance curves for M = 2,4,10, and 26.

When M=2, Pc probability that a standard normal variable z0 with mean 0 and variance 1 is less that an independent standard normal variable zd with mean δ and variance 1, which is

$$\Pr(\text{Correct}) = \Pr((zd - z0 > 0) = Fz(\delta/\sqrt{2}). \quad (40)$$

The Yes/No detection Equation 15 implies that

$$d' = Fz^{-1}(\Pr(\text{Correct})) - Fz^{-1}(1 - \Pr(\text{Correct}))$$
$$= \delta/\sqrt{2} - (-\delta/\sqrt{2}) = \sqrt{2}\,\delta. \quad (41)$$

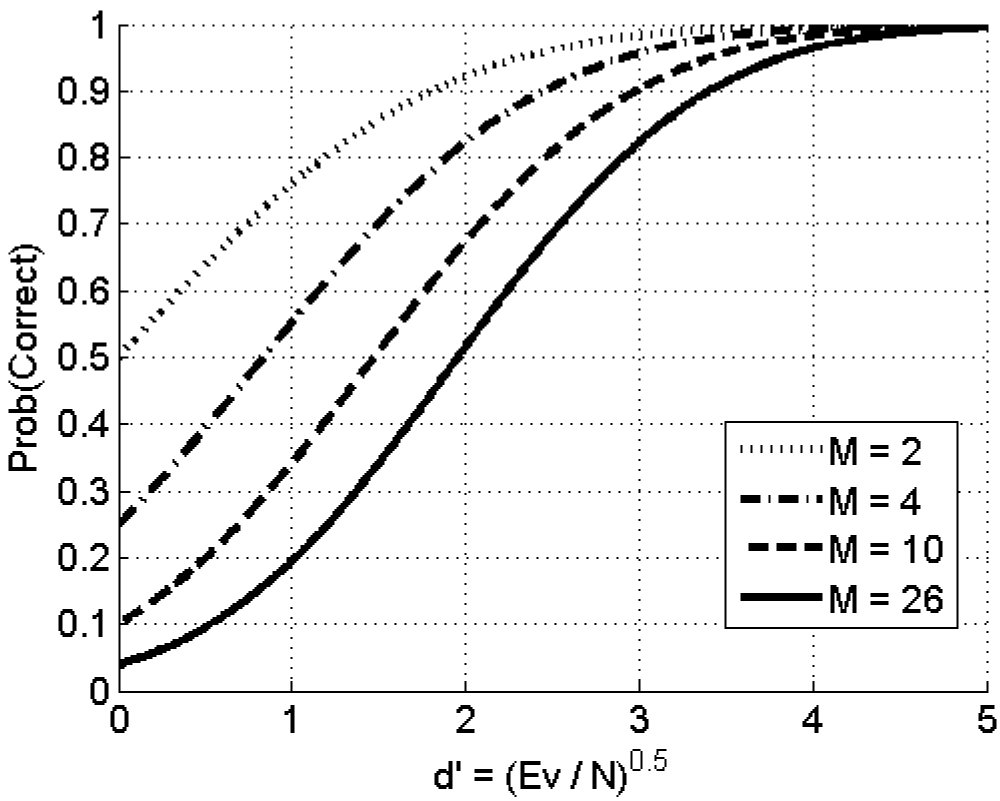

Figure 1. Probability of a correct discrimination for the orthogonal, equal energy visible contrast image model.

A general discrimination metric

Dalimier and Dainty [9] have proposed basing a general discrimination metric on the total squared distance from each visible contrast pattern to the average of the M patterns. If we let

$$Av = \Sigma j\ Cvj\ /M, \quad (42)$$

Their metric is

$$d'^{2} = (4/(\sigma^{2} M))\ \Sigma j\ ||Cvj - Av||^{2}. \quad (43)$$

They introduced the 4 so that when M = 2, the formula would reduce to the usual Yes/No d' formula. When M=2, the mean of any two points is halfway between them, $||Cv1–Av|| = ||Cv1–Av||= ||Cv1–Cv2||/2$, so

$$d'^{2} = (4/(\sigma^{2} 2))\ 2\ ||Cv1–Cv2||^{2}/4 = ||Cv1–Cv2||^{2}/\sigma^{2}. \quad (44)$$

As shown in Appendix B, the total of the squared distances among all M^2 pairs is 2 M times the total of the squared distances from the patterns to the mean pattern,

$$\Sigma j,k\ ||Cvj - Cvk||^{2} = 2\ M\ \Sigma j\ ||Cvj - Av||^{2}. \quad (45)$$

If we do not count the distances from each point to itself the actual number of distances among points is $M^2 – M$, the average distance among the points is

$$(1/(M(M–1))\ \Sigma j,k\ ||Cvj - Cvk||^{2} = (2/(M–1))\ ||Cvj - Av||^{2}. \quad (46)$$

That is, the average distance among the points is twice the average distance of each point to the mean when M–1, the number of linearly independent distances, is used in the denominator.

If Dalimier and Dainty [9] had started with the average distance among the points, the formula would have needed no correction factor

$$d'^{2} = (1/(M\ (M–1)\ \sigma))\ \Sigma j,k\ ||Cvj - Cvk||^{2} = (2/(\sigma\ (M–1))\ \Sigma j\ ||Cvj–Av||^{2} \quad (47)$$

which needs no correction for M=2.

Another metric which has been used [10] for comparing alphabets is the average correlation among the patterns. If we define the average of vector Cvk to be Avk, and Dvk to be Cvk – Avk, the average correlation ρ is

$$\rho = (1/M(M–1))\ \Sigma_{j \neq k}\ Dvj)\bullet(Dvk\ /(||Dvj||\ ||Dvk||). \quad (48)$$

If the patterns have equal lengths, L =||Cj|| and equal means A = Avj, they also have equal lengths about their own mean, V, since

$$V^{2} = ||\ Dvk||^{2} = ||Cvj - Avj|| = ||Cj||^{2} - \text{ave}(Cj)^{2} = L^{2} - A^{2}. \quad (49)$$

In this case, as shown in Appendix B,

$$\rho = 1 - (1/((M–1)\ V^{2}))\ \Sigma j\ ||Cj - C||^{2}. \quad (50)$$

Again, the natural divisor for the average squared distance to the mean is M–1.

In the general orthogonal case, as shown in Appendix B,

$$(1/(M–1)\ \Sigma j\ ||Cv(j) - Cv||^{2} = (1/M)\ \Sigma j\ ||Cv(j)||^{2}. \quad (51)$$

If the lengths are also equal,

$$(1/(M–1))\ \Sigma j\ ||Cv(j) - Cv||^{2} = (1/M)\ \Sigma j\ L^{2} = L^{2}. \quad (52)$$

Corresponding to Equation 8 for detection, a resolution-independent metric for discrimination is given by

$$Ev = dx\ dy\ dt\ (1/(M–1))\ \Sigma j\ ||Cv(j) - Cv||^{2}. \quad (53)$$

When used with Equations 38 and 39 (Figure 1) it correctly predicts discrimination performance for equally likely alternatives when M =2 and should be useful both for finding starting values for N in model simulations and providing a way of equating results from studies using different values for the Pr(Correct).

Example 1: Landolt C's

Figure 2 shows a rectangular version of the familiar Landolt C pattern. Since the distances among the patterns are unchanged by adding or subtracting a constant pattern from all M patterns, the ideal observer is unaffected by such a manipulation. If an unbroken square is subtracted from all the patterns, the result is 4 points that are non-overlapping, and thus orthogonal. Performance on this pattern set is thus predictable from Equation 26 and Figure 1.

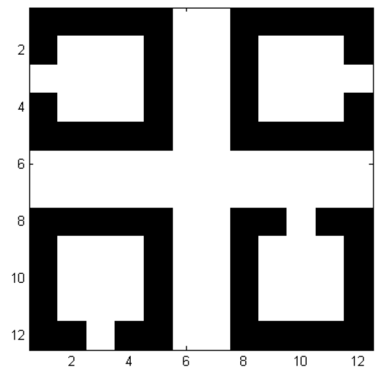

Figure 2. Rectangular Landolt C patterns.

Example 2: Tumbling E

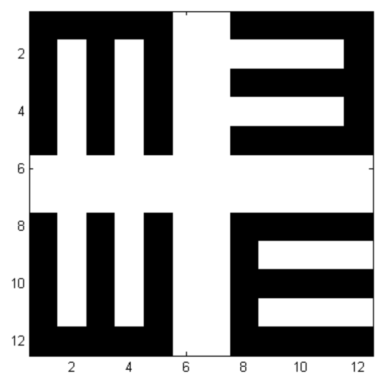

Figure 3. Tumbling E patterns.

Figure 3 shows four Tumbling E patterns. Each pattern has 17 pixels, so they have equal energy. The pixel distances between the reflected versions are 4 and the rotated versions are 8. These patterns are not orthogonal, but do have equal energy. Equation (24) evaluated at M=4 and d'=1 results in the prediction Pc = 0.552 Model simulation for 10,000 trials led to the 95% confidence interval for Pc = 0.538 ± 0.098, which is statistically significantly lower, but usefully close. Appendix A shows the Matlab code for the simulation and the code for computing Pc.

Estimating N from discrimination

Watson and Ahumada[2] used simulation to fit the model (their model ID) to Sloan letter acuity data (M=10) and generated estimated values of σ in decibels of contrast for four observers (-0.5, -1, -2.2, -4.7). Their pixel resolution was 313.91 pixels/deg. Assuming a fixation duration of 0.25 sec gives an additive constant for estimating N as $60 - 20\log_{10} 313.91 + 10\log_{10} 0.25 = 10.1$ dB. The median of the four estimates is 8.5 dBB. Less ideal observer models gave smaller estimates for N. Model XA allowed for spatial uncertain and ignored energy (used cross correlation alone) and gave an N estimate of 5.8 dBB. Model XL with spatial uncertainty and templates based on the unfiltered letters gave an estimate of -1.3 dBB, closer to the estimate from detection.

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Appendix A: Matlab Code

Contrast image to visible contrast image

```
function cv = con2convis(duration,pixperdeg,contrast_image)
im = contrast_image;
n = size(im,1) ;
ppd = pixperdeg;
f0 = 12; f1 = 2; % cpd
a = 1 - exp(-duration) ;
csf = fltexp2(n,f0*n/ppd) - a*filtgaus2(n,f1*n/ppd);
cv = real(ifft2(fft2(im).*csf)) ;
cv = cv.*curcio2(n, ppd) ;

function cv = lum2convis(duration,pixperdeg,lum_image, background)
im = lum_image;
n = size(im,1) ;
ppd = pixperdeg;
f0 = 12; f1 = 2; % cpd
a = 1 - exp(-duration) ;
ocsf = fltexp2(n,f0*n/ppd);
bcsf = ocsf.*filtgaus2(n,f1*n/ppd);
imo = real(ifft2(fft2(im).*ocsf)) ;
imb = real(ifft2(fft2((1-a)*background+a*im).*bcsf)) ;
cv = imo./imb -1;
cv = cv.*curcio2(n, ppd) ;

% Subroutines

function filter = fltexp2( n, f)  % Exponential
  filter = exp(-fltf2(n)/f) ;

function  flt = filtgaus2(n, f)   % Gaussian
```

```
  flt1 = fltgaus1(n,f);
  flt=flt1'*flt1;

function filter = fltgaus1( n, f) % 1-D Gaussian
  filter = fltf1(n)/f;
  filter = exp(-(filter.*filter))  ;

function f = fltf2(n)      % 2-D frequencies in cycles per image
  f1 = fltf1(n);
  f = repmat(f1.*f1,[n 1]) ; % fx^2
  f = sqrt(f + f') ;         % sqrt(fx^2 + fy^2)

function f = fltf1( n)      % 1-D frequencies in cycles per image
  n2 = ceil(n/2);
  f = [[0:floor(n/2)] [1:n2-1]-n2];

function window = curcio2(n,ppd)
% windowing function from Ahumada and Watson (2013)
% centered at n/2 + 1 if n even , 1+floor(n/2) if n is odd
xe = 5.72;
g = 4.09;
x = ([1:n]-(1+floor(n/2)))/ppd; % degrees
x = repmat(x.*x,[n 1]) ;
x = sqrt(x + x');
window = 1 ./ (1+g*(1-exp(-(x / xe)))) ;
```

Tumbling E Discrimination Model Simulation

```
s = sqrt(0.3)*[ % normalized so that d = 1
     [1 1 1 1 1 ...
      1 0 0 0 0 ...
      1 1 1 1 1 ...
      1 0 0 0 0 ...
      1 1 1 1 1 ] ; ...
     [1 1 1 1 1 ...
      0 0 0 0 1 ...
      1 1 1 1 1 ...
      0 0 0 0 1 ...
      1 1 1 1 1 ] ; ...
     [1 0 1 0 1 ...
      1 0 1 0 1 ...
      1 0 1 0 1 ...
      1 0 1 0 1 ...
      1 1 1 1 1 ] ; ...
     [1 1 1 1 1 ...
      1 0 1 0 1 ...
      1 0 1 0 1 ...
      1 0 1 0 1 ...
      1 0 1 0 1 ] ]';
s = s - repmat( mean(s,2),[1 4]) ;
d = sum(sum(s.*s))/(size(s,2)-1) ;% 1
ss = s'*s ; % k x k
[u , x, v] = svd(ss) ;
```

```
f = u*(x.^0.5) ;
cor = repmat(ss(1,:),[n 1])+randn(n,k)*f' ;
Pc = mean(cor(:,1) > max(cor(:,2:k)')');
% 0.5383 % plus or minus 1.96 *sqrt(0.5*0.5/10000) = 0.01
```

Pc for orthogonal equal energy visible contrast images (Equation (25))

```
M = 4; d = 1;
F = @(x)exp(-0.5*x.*x).*(1+erf((x+1)/1.41421356237310)).^(M-1);
Pc = quad(F,-4,4)*0.5^(M-1)/sqrt(2*pi); % 0.5518
```

Appendix B: Derivations

Equation 45

The total of the squared distances among all M^2 pairs is

$$\begin{aligned}
&\Sigma_{j,k} \|C_j - C_k\|^2 \\
&= \Sigma_{j,k} ((C_j - C_k)\bullet(C_j - C_k)) \\
&= \Sigma_{j,k} (C_j\bullet C_j + C_k\bullet C_k - 2\, C_j\bullet C_k) \\
&= M\, \Sigma_j C_j\bullet C_j + M\, \Sigma_k C_k\bullet C_k - 2\, \Sigma_{j,k} C_j\bullet C_k \\
&= 2\, (M\, \Sigma_j C_j\bullet C_j) - \Sigma_{j,k} C_j\bullet C_k.
\end{aligned} \tag{B1}$$

The sum of the squared distances to the mean is

$$\begin{aligned}
&\Sigma_j \|C_j - C\|^2 \\
&= \Sigma_j ((C_j - C)\bullet(C_j - C)) \\
&= \Sigma_j (C_j\bullet C_j + C\bullet C - 2\, C\bullet C_j) \\
&= \Sigma_j C_j\bullet C_j + \Sigma_j C\bullet C - 2\, \Sigma_j C\bullet C_j \\
&= \Sigma_j C_j\bullet C_j - M\, C\bullet C \\
&= \Sigma_j C_j\bullet C_j - (1/M)\, (\Sigma_j C_j)\bullet(\Sigma_k C_k) \\
&= \Sigma_j C_j\bullet C_j - (1/M)\, \Sigma_{j,k} C_j\bullet C_k.
\end{aligned} \tag{B2}$$

Thus

$$\Sigma_{j,k} \|C_j - C_k\|^2 = 2\, M\, \Sigma_j \|C_j - C\|^2 \tag{B3}$$

Equation 50

$$\begin{aligned}
\rho &= (1/(V^2 M(M-1)))\, (\Sigma_{j \neq k} (C_j - A)\bullet(C_k - A)) \\
&= (1/(V^2 M(M-1)))\, (\Sigma_{j,k} (C_j - A)\bullet(C_k - Ckbar) - \Sigma_j \|C_j - A\|^2) \\
&= (1/(V^2 M(M-1)))\, (\Sigma_{j,k} (C_j.C_k) - M^2 A^2 - M\, V^2).
\end{aligned} \tag{B4}$$

From Eq 27 we see that

$$\Sigma_{j,k} C_j.C_k. = M\, (\Sigma_j C_j.C_j - \Sigma_j \|C_j - C\|^2) = M^2 L^2 - M\, \Sigma_j \|C_j - C\|^2$$

So that

$$\begin{aligned}
\rho &= (1/(V^2 M(M-1)))\, (M^2 L^2 - M\, \Sigma_j \|C_j - C\|^2 - M^2\, Cbar^2 - M\, V^2) \\
&= (1/(V^2 (M-1)))\, (M\, L^2 - \Sigma_j \|C_j - C\|^2 - M\, Cbar^2 - V^2) \\
&= (1/(V^2 (M-1)))\, (M\, V^2 - \Sigma_j \|C_j - C\|^2 - V^2) \\
&= (1/(V^2 (M-1)))\, ((M-1)\, V^2 - \Sigma_j \|C_j - C\|^2) \\
&= 1 - (1/(V^2 (M-1)))\, \Sigma_j \|C_j - C\|^2 .
\end{aligned} \tag{B5}$$

Equation 51

$$\begin{aligned}
\Sigma_j \|Cv(j) - Cv\|^2 &= \Sigma_j(\|Cv(j)\|^2 - (\Sigma_j Cv(j))(\Sigma_k Cv(k))/ M^2 \\
&= \Sigma_j \|Cv(j)\|^2 - \Sigma_j \Sigma_k Cv(j)\bullet Cv(k)/M^2 \\
&= \Sigma_j \|Cv(j)\|^2 - \Sigma_j\ Cv(j)\bullet Cv(j)/M^2
\end{aligned}$$

$$= \Sigma_j \|Cv(j)\|^2 - \Sigma_j \|Cv(j)\|^2)/M$$
$$= (1 - 1/M)\, \Sigma_j \|Cv(j)\|^2 . \qquad \text{(B6)}$$

Thus

$$(1/(M-1))\, \Sigma_j \|Cv(j) - Cv\|^2 = (1/M)\, \Sigma_j \|Cv(j)\|^2 . \qquad \text{(B7)}$$