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Combining Speed Information Across Space

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We used speed discrimination tasks to measure the ability of observers to combine speed information from multiple stimuli distributed across space. We compared speed discrimination thresholds in a classical discrimination paradigm to those in an uncertainty/search paradigm. Thresholds were measured using a temporal two-interval forced-choice design. In the discrimination paradigm, the *n* gratings in each interval all moved at the same speed and observers were asked to choose the interval with the faster gratings. Discrimination thresholds for this paradigm decreased as the number of gratings increased. This decrease was not due to increasing the effective stimulus area as a control experiment that increased the area of a single grating did not show a similar improvement in thresholds. Adding independent speed noise to each of the *n* gratings caused thresholds to decrease at a rate similar to the original no-noise case, consistent with observers combining an independent sample of speed from each grating in both the added- and no-noise cases. In the search paradigm, observers were asked to choose the interval in which one of the *n* gratings moved faster. Thresholds in this case increased with the number of gratings, behavior traditionally attributed to an input bottleneck. However, results from the discrimination paradigm showed that the increase was not due to observers' inability to process these gratings. We have also shown that the opposite trends of the data in the two paradigms can be predicted by a decision theory model that combines independent samples of speed information across space. This demonstrates that models typically used in classical detection and discrimination paradigms are also applicable to search paradigms. As our model does not distinguish between samples in space and time, it predicts that discrimination performance should be the same regardless of whether the gratings are presented in two spatial intervals or two temporal intervals. Our last experiment largely confirmed this prediction.

Multiple stimuli Integration Search Uncertainty Speed discrimination

INTRODUCTION

Vision scientists have made significant progress in understanding the detection of single stimuli at threshold, but much less is known about how we process complex scenes consisting of multiple stimuli. The issue of how we combine information from individual stimuli to get the big picture continues to be an important, albeit difficult problem. At present, there is considerable psychophysical and physiological evidence that the visual system decomposes the visual scene using local mechanisms tuned for specific stimulus properties such as spatial frequency, orientation, direction of motion etc. (De Valois & De Valois, 1988). However, there is still not a clear understanding of how all of these different local components are recombined to synthesize an apparently seamless visual scene. In this study we approach this problem by working at a level that is intermediate in complexity between single stimuli and complex scenes, i.e. at the level of processing multiple discrete stimuli. Our interest is in the ability of human observers to combine information from multiple stimuli across space. Specifically, we would like to know if the ability to process multiple stimuli can be predicted from what is known about the processing of single stimuli. To this end, we have extended the methods of classical psychophysics used in the study of single stimuli to the issue of integrating information from multiple stimuli.

The issue of stimulus integration in classical psychophysics has been addressed largely by summation experiments. In these experiments, the observer's threshold for a compound stimulus is compared to thresholds for the component stimuli that make up the compound. When the compound is more detectable than either of the components, the results have been explained by summation within a mechanism that is sensitive to more than one of the components, or by probability summation between independent mechanisms that are each sensitive to only one of the components. For the case of detecting a single stimulus as a function of its spatial extent, the compound can be thought of as made of several identical, spatially juxtaposed stimuli. Detection thresholds support summation within a mechanism for small stimuli, and probability summation across space between independent detectors for extended stimuli (e.g. King-Smith & Kulikowski, 1975; Legge, 1978; Robson &

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Graham, 1981; Wilson, 1978). One of our goals is to examine if a simple model that combines information from independent detectors, similar to those that predict summation at detection threshold, can explain the processing of multiple discrete suprathreshold elements distributed across space.

The processing of multiple stimuli has been of particular interest to scientists studying visual attention. Attention has been regarded as having a limited input capacity, which restricts the amount of information an observer can absorb in a brief period of time (Broadbent, 1958; Neisser, 1967). The effects of this bottleneck have been studied using visual search experiments in which observers are presented with multiple stimuli (Treisman & Gelade, 1980; Bergen & Julesz, 1983). The task in search experiments is to detect the presence or absence of an odd element in a background consisting of similar elements. These studies show that the probability of a correct response at a fixed presentation duration decreases as the number of background elements is increased. This result has typically been interpreted in terms of the limits of visual attention; rarely have they been related to the results from classical psychophysics on the processing of single stimuli (however, see Pavel, Econopouly & Landy, 1992; Palmer, Ames & Lindsay, 1993; Verghese & Nakayama, 1994.). Visual search is a subset of paradigms using multiple stimuli and we would like to determine if the processing in multiple-stimuli paradigms can be understood as an extension of what is known about the processing of single stimuli.

We have selected paradigms that reflect both of the above approaches, i.e. threshold psychophysics and visual-search methodologies. We use moving grating patches as our experimental stimuli, as much is known about the early mechanisms involved in the extraction of local motion information. Yet, it is clear that some way of combining local signals such as these would be necessary for motion-based image segmentation or object recognition. We use these paradigms in the context of a speed-discrimination experiment. Our first paradigm is an extension of a standard discrimination task to multiple stimuli. The stimulus consists of two temporal intervals, each with n gratings, with all the gratings in one interval moving faster than the gratings in the other. The observer's task is to choose the interval with the faster gratings. Figure 1(a) shows a schematic of this experiment. Our second paradigm has aspects in common with both uncertainty and visual-search experiments. In this paradigm only one of the n gratings in one of the intervals moves faster. The observer's task is to choose the interval with the faster grating. In this case increasing the number of gratings in an interval increases the spatial uncertainty of the faster grating. Alternatively, this paradigm can be thought of as a search task in which increasing the number of gratings in an interval increases the number of distractor gratings. Figure 1(b) shows a schematic of the uncertainty experiment. These two paradigms differ in only one key aspect—the number of gratings that are moving faster. Our study demonstrates that this single difference causes opposite trends in the thresholds of the two paradigms. Explicit simulations of simple decision models that combine independent speed estimates from each of the multiple gratings predict both of these trends. Our results provide strong empirical as well as theoretical evidence against performance in these experiments being constrained by limited-capacity mechanisms (e.g. Treisman & Gelade, 1980).

Another important issue in the processing of multiple gratings is the effect of stimulus paradigm. Previous studies of speed discrimination presented two gratings either in two temporal or two spatial intervals (e.g. Diener, Wist, Dichgans & Brandt, 1976; Thompson, 1982; McKee, Silverman & Nakayama, 1986; Ferrara & Wilson, 1991; Smith & Edgar, 1990; Stone & Thompson, 1992; Hawken, Gegenfurtner & Tang, 1994). Simple signal detection models predict that these two presentation paradigms should have no differential effect on speed discrimination. However, the aforementioned studies have shown contradictory results for different stimulus presentation paradigms, suggesting that sequential as opposed to simultaneous presentation might affect speed discrimination thresholds.* Our third experiment addresses this issue specifically and shows that sequential vs simultaneous presentation *per se* has no effect on speed discrimination threshold. This experiment provides independent confirmation of the fundamental assumption of our model that observers use independent speed estimates from each grating.

METHODS

Our experiments measured speed discrimination, using a two-interval forced-choice design. Observers were presented with multiple moving gratings in two temporal intervals, unless otherwise stated. Both intervals had the same number of gratings, n, varying from 1 to 6. In addition to the three main experiments we outlined in the Introduction, we also conducted four controls for the first experiment. Methodological details specific to each experiment are provided in the section dealing with that experiment.

Observers

Four observers with normal acuity participated in our experiments; three were experienced in psychophysical experiments, the other (ET) was naive as to the purpose of our experiments and a novice observer. As our naive observer showed significant improvement over time, the data that we have plotted here were measured after considerable practice, after her thresholds had decreased to the same range as the experienced psychophysical observers. We have partial data from two additional

*Johnson and Leibowitz (1974) have shown that foveal vs peripheral presentation of the stimuli as well as the use of feedback are important factors in determining direction of motion thresholds. Given that sequential vs simultaneous presentation is highly correlated with foveal vs peripheral presentation, this is often a confounding issue.

observers. Their data (not shown) are qualitatively similar to the data presented in this paper.

Stimuli

Stimuli were displayed on a 12 in. Apple high-resolution monochrome monitor driven by an 8-bit Macintosh Display Card installed in a Quadra 900. The monitor had a resolution of 30 pixels/deg at a viewing distance of 57.5 cm. The non-linear gamma function of the monitor was measured and used for accurate control of the luminance of the display (Pelli & Zhang, 1991). The background luminance of the display was 20 cd/m^2 and observers viewed the display binocularly.

The stimuli were Gabor patches, i.e. moving sinusoidal gratings windowed by a stationary two-dimensional spatial Gaussian with SD of 0.4 deg in the x and y dimensions. The phase of the grating with respect to the center of the Gaussian window was randomized. The number of these Gabor patches, n, was either 1, 2, 4, or 6, but kept fixed within a block of trials. The n gratings presented in an interval were distributed uniformly around a concentric ring of eccentricity 4 deg. Figure 1 illustrates the display with six gratings. The spatial frequency was fixed at 1.5 c/deg, and the orientation was always vertical. Speed discrimination was measured by varying temporal frequency, about a reference near 8 Hz. To minimize the possibility of observers basing their decision on the speed in a single interval, the reference temporal frequency was jittered in seven steps between 8 ± 1 Hz. Contrast was fixed at 50%.

Unless otherwise stated, the position of the gratings in each interval was determined by placing the first grating at one of 12 clock positions (at 30 deg intervals), and the remaining $(n - 1)$ gratings at equal spatial intervals of 360 deg/n. The position of the first grating was randomly chosen. Unless otherwise stated, all the gratings in an interval moved in the same direction, either all left or all right, and this direction was selected randomly for each interval.

The moving stimuli were created by precomputing each frame and storing the stimuli on disk. Each trial started with a warning beep, followed by the first interval, a blank duration of 500 msec, and finally the second interval. Each interval was a movie of 13 frames, which lasted 195 msec. (The monitor's frame rate was 66.7 Hz.) The intervals were brief in order to minimize eye movements. After the second interval, observers were asked to press one of two keys, depending on whether they thought the faster grating was in the first or the second interval. Feedback was provided to all observers except LS. The next trial started 1 sec after the response. Observers were asked to fixate a cross that was on during the entire block. They were also asked to hit a "redo" button if they broke fixation or if they blinked during stimulus presentation. We used a 3 up–1 down staircase to control the speed difference between intervals. The staircase terminated after 12 reversals. A session consisted of four blocks, each with 1, 2, 4, or 6 gratings. The order of the blocks was randomized in each session. At least four sessions were run for each condition. The raw data for each condition and number of gratings was pooled over all runs and fit with a Weibull function. The fitting procedure minimized the χ^2 of the fit to the data. The χ^2 was computed by weighting the data points by their SDs, assuming a binomial distribution. The threshold was determined from this fit and taken as the speed difference corresponding to 82% correct.

RESULTS

Experiment 1

Multiple-grating speed discrimination

Method. Observers were presented with two temporal intervals, each with n gratings, with all the gratings in one interval moving faster than the gratings in the other [Fig. 1(a)]. Their task was to choose the interval with the faster gratings. The number of gratings varied from 1 to 6, and these were presented in random locations.

Results. Figure 2 plots the raw data for one observer. Proportion correct is plotted vs the speed difference between the two intervals with the different symbols indicating the number of gratings. The lines are the best Weibull fits to the raw psychometric data. These data show that as the number of gratings is increased, the curves shift to the left, indicating that threshold decreases, at least up to four grating patches for this observer. In the subsequent figures we do not plot the individual data points; instead we summarize each psychometric curve by its threshold, the 82% correct point. Figure 3 summarizes the decrease in threshold with number for all of our observers. The threshold speed difference is plotted versus the number of grating patches. The error bars represent ± 1 SD of the estimate of threshold. (In order to avoid overlap between error bars, we plot error bars only for our naive observer, ET, whose data had the highest variance.) The improvement shows that observers use information provided by multiple gratings in making their decision.

Control for spatial uncertainty

One possible explanation for why threshold decreases as the number of gratings is increased is that the observer is uncertain about the location of a single grating and that increasing the number of gratings, increases the probability that a grating appears in a location that the observer is randomly monitoring. In other words, increasing the number of gratings reduces the effective spatial uncertainty and thresholds improve merely because of this. We therefore did the following control.

Method. We eliminated spatial uncertainty by always placing the gratings at known locations. When a block of trials consisted of a single grating it always appeared directly to the right of fixation, i.e. at the 3 o'clock position. When the block consisted of $n > 1$ gratings, the first grating appeared directly to the right, the others at a spacing 360 deg/n away, thus eliminating spatial uncertainty for all values of n. We also did not jitter the reference speed in this control experiment.

Results. Figure 4(a) shows data for all four observers for the fixed-position, no-jitter control. Discrimination thresholds again decrease as the number of grating patches is increased. Absolute thresholds are roughly 30% lower for all grating numbers than in the original paradigm with jitter and randomly-positioned gratings (cf. Fig. 3). If the improvement in absolute threshold were largely due to eliminating spatial uncertainty, then the benefit of reducing uncertainty should decrease as a function of n, the number of gratings, causing thresholds in the two cases to converge as n is increased. (Spatial uncertainty would be zero for $n = 12$ in both the known- and unknown-location conditions, as there were 12 possible "random" locations.) A comparison of Figs 3 and 4(a) shows that such a convergence is not seen. Specifically, if uncertainty played a major role, thresholds should decrease at a slower rate in the known-position case than in the unknown-position case. This does not happen, as illustrated in Fig. 4(b). The solid and open symbols represent relative thresholds for gratings in unknown and known locations respectively. Each observer's threshold was normalized to that for a

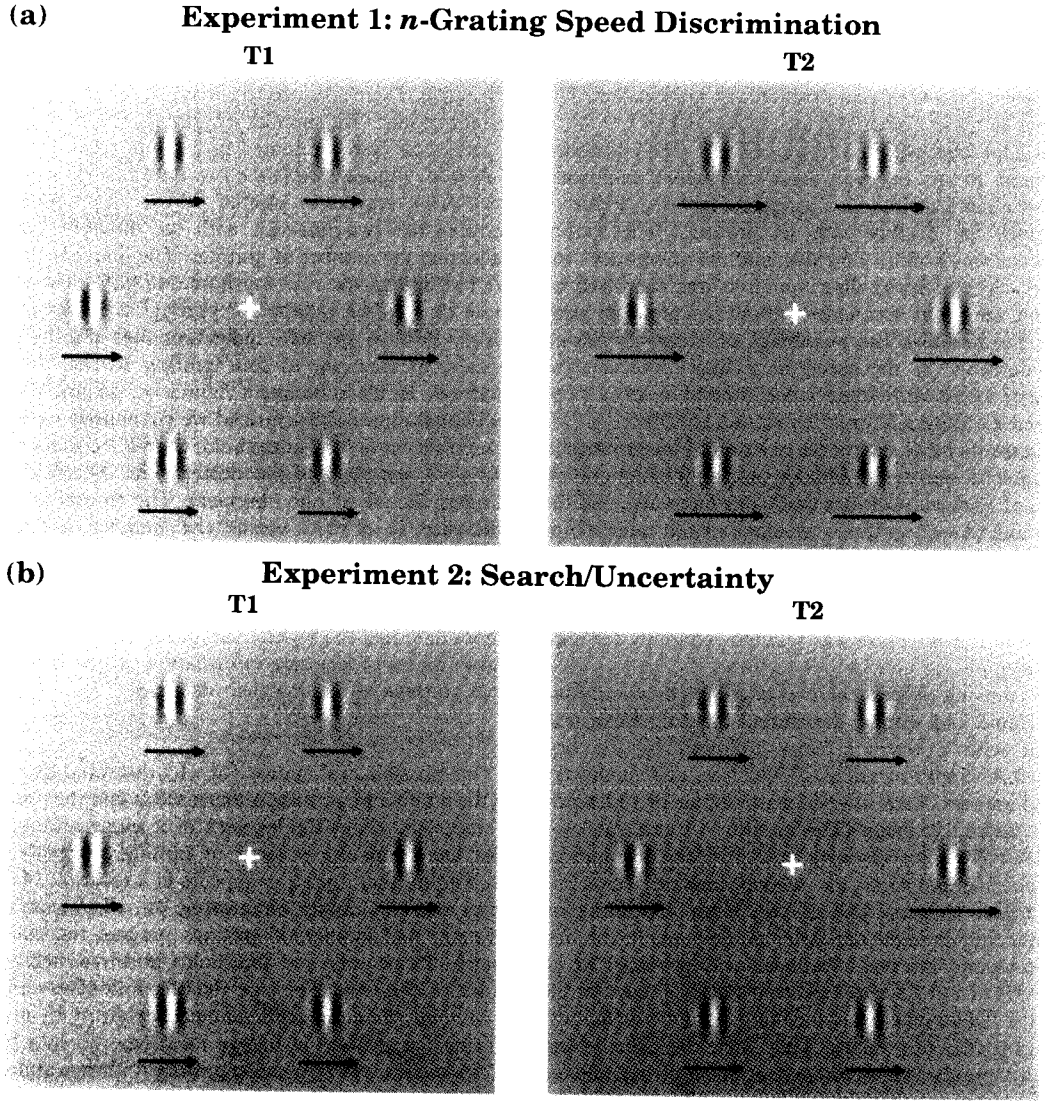

FIGURE 1. Static frames of the stimulus, showing six sinusoidal grating patches windowed by a circularly symmetric Gaussian. The patches were presented in a ring at an eccentricity of 4 deg from fixation. The two panels correspond to the two temporal intervals, T1 and T2, each with n gratings. The black arrows are for purposes of illustration alone and their lengths represent the speed of the gratings. (a) In Expt 1 all the gratings in an interval moved at the same speed and observers were asked to choose the interval with the faster gratings. (b) In Expt 2 one of the gratings in one of the intervals moved faster, while all the remaining gratings moved at the same speed. Observers were asked to choose the interval with the faster grating.

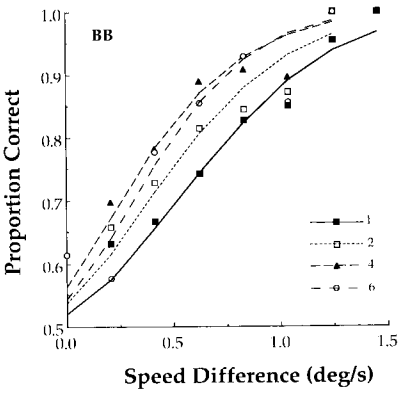

FIGURE 2. Psychometric functions for observer BB, in Expt 1. Proportion correct is plotted vs threshold speed difference for different numbers of gratings: 1 (■), 2 (□), 4 (▲) and 6 (○). The lines through the points are the best-fitting Weibull function to the data. The average reduced χ^2 of the fits was 0.87.

single grating, and the data points of Fig. 4(b) are the average of normalized thresholds for all observers. Relative thresholds decrease with the number of gratings at comparable rates for gratings in known and unknown locations.

To further confirm that the improvement in absolute thresholds was not due to the lack of spatial uncertainty, observer PV repeated the speed discrimination experiment with no speed jitter and with gratings in random positions. In this case the absolute thresholds (not shown) were comparable to her data for the no-jitter, fixed-position case [Fig. 4(a)] and showed the same trend with number. The fact that absolute thresholds in the known- and unknown-location case are similar in the absence of speed jitter, and that they are lower than thresholds measured in the presence of jitter, suggests that it is the lack of jitter rather than knowledge of

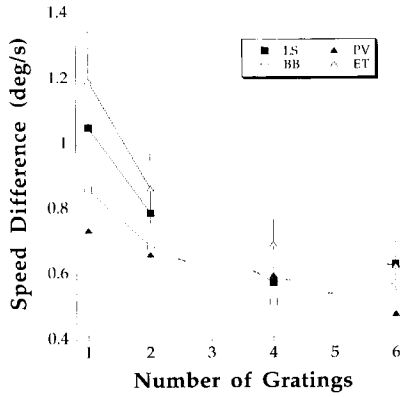

FIGURE 3. Threshold speed difference as a function of the number of grating patches for Expt 1. The different curves are for different observers as indicated in the figure legend. The error bars represent ± 1 SD of the estimated thresholds of our naive observer, ET, whose data in this and subsequent experiments typically had the largest variance.

grating location that causes the difference in absolute thresholds between Figs 3 and 4(a). The results of these control experiments show that the large decrease in threshold with number in the original data of Fig. 3 is unlikely to be due to the reduction of spatial uncertainty with number.

Controls for stimulus area and relative position

A logical question at this point is whether the improvement in threshold seen in Expt 1 is simply due to the increase in the area of stimulation as the number of gratings is increased. Could this be responsible for the decrease in thresholds as in the case of grating detection (Robson & Graham, 1981)? To control for effective stimulus area, we did the following experiment.

Method. We used a single grating in each interval, equal to 1, 2, 4 or 6 times the area of the original grating. This grating was always presented at a random location, centered 4 deg from fixation. A single grating with n times the area of the original was created by increasing the standard deviation of the Gaussian window in the x and y dimensions by $\sqrt{n}$.

Results. Speed difference is plotted vs the area of the single grating for all four observers in Fig. 5(a). Increasing grating area has little if any effect on thresholds. Fig. 5(b) plots normalized thresholds (relative to that for one grating), averaged across all four observers as a function of grating area. The solid symbols depict average data for the original multiple-grating experiment whereas the open symbols depict average data for the single-grating area control experiment. Thresholds decrease as the number of gratings is increased, but remain roughly constant when the area of a single grating is increased.

It is possible that thresholds decrease in the original multiple-grating case because the gratings were in different spatial locations and that thresholds would not decrease similarly if the multiple gratings were placed in adjacent positions. The rationale is that when the multiple gratings are clustered, their spatial uncertainty would be equivalent to that of a single grating. To address this issue further, we repeated the original two-grating paradigm using two spatial configurations. In addition to the original version with the two gratings diametrically opposite, we also measured thresholds for the condition in which the two gratings occupied adjacent positions 30 deg apart (almost touching). Figure 6 shows that speed discrimination thresholds measured with the gratings in these two configurations were indistinguishable. Paired t-tests on the thresholds for these two configurations are not significant [$T(3) = 0.41$; $P = 0.71$] while the decrease in thresholds from 1 to 2 gratings in Fig. 3 is significant [$T(3) = 3.77$; $P = 0.03$]. These results, along with the fact that increasing the area of a single grating has no effect, suggest that increasing the number of gratings from one (regardless of its size) to two (regardless of their spacing) causes thresholds to decrease. Therefore, it appears that it is the multiple discrete grating patches of the original experiment that cause thresholds to decrease.

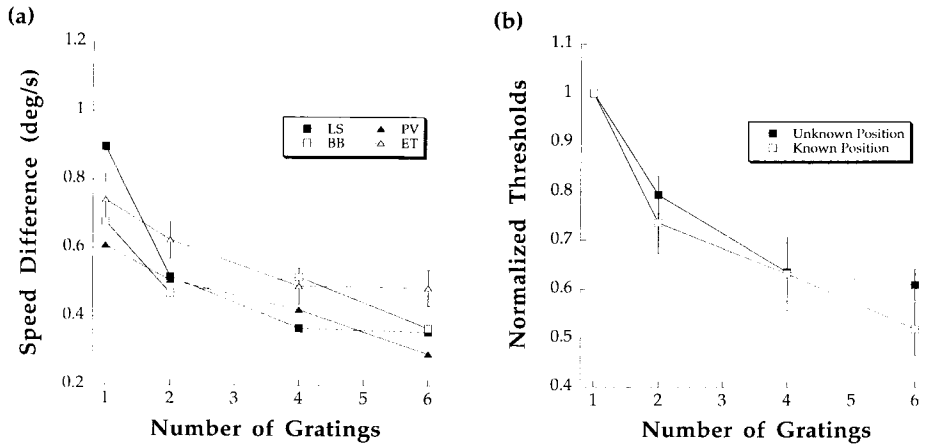

FIGURE 4. (a) Threshold speed difference as a function of the number of grating patches in the control for spatial uncertainty. The error bars represent ±1 SD of the threshold. (b) Thresholds normalized to that for one grating and averaged across observers are plotted as a function of the number of grating patches. The error bars represent ±1 SE across observers. The open symbols are for the case when stimulus position was known [data from (a)], and the solid symbols for the case when stimulus position was random (Fig. 3).

Control for correlated noise

To understand the lack of an effect of increasing stimulus area in the area control experiment, as opposed to the multiple-grating experiment, let us assume that the visual system gets at least one estimate of speed from each grating patch. Thresholds in the multiple-grating case would improve with number if these estimates were independent. Observers would thus benefit from combining the multiple speed estimates from the discrete grating patches. In the case of the area control experiment, one explanation for why thresholds do not improve with area is that the multiple estimates from a single large grating might be highly correlated (non-independent). In this case pooling multiple samples from the same grating would produce little benefit. In fact, it is possible that even the speed estimates from the discrete grating patches in the multiple-grating paradigm were partly correlated. Such a correlation could be reduced by adding large amounts of independent speed noise to each of the gratings. Adding independent noise would thus cause *relative* thresholds as a function of number to improve at a faster rate. Of course, the *absolute* thresholds would increase monotonically with the amount of added speed noise.

Method. We tested this independent-samples hypothesis by modifying the original experiment so that the grating speeds were indeed independent samples the gratings in each interval came from a Gaussian distribution of speed, centered about the mean speed of each interval, with a SD of 1.8 deg/sec. This SD was about 3 times the speed discrimination threshold for a single

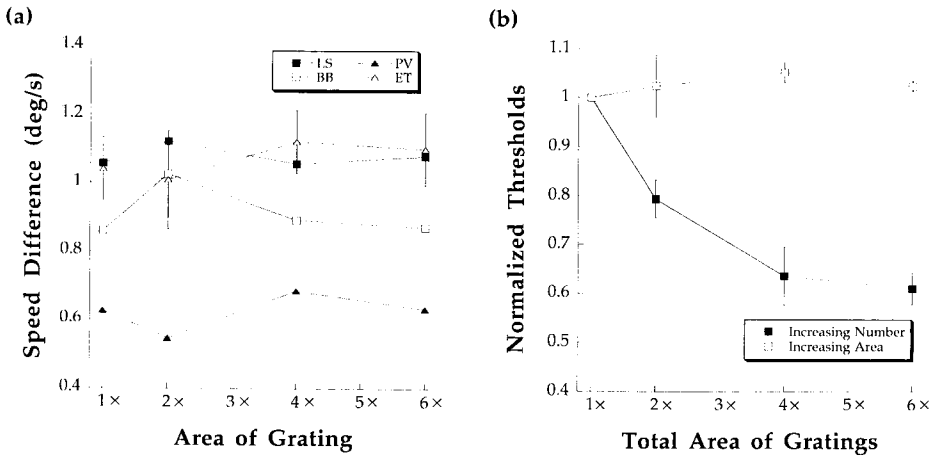

FIGURE 5. (a) Threshold speed difference as a function of the area of a single grating. The error bars represent ±1 SD of the threshold. (b) Thresholds normalized to that for one grating and averaged across observers are plotted against total grating area. The error bars represent ±1 SE across observers. The open symbols are for the case when the area of a single grating was increased [data from (a)], and the solid symbols for the case when the effective stimulus area was increased by increasing the number of gratings (Fig. 3).

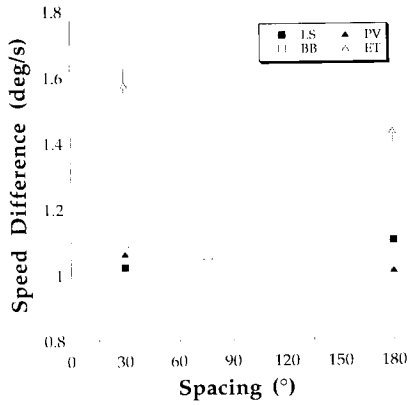

FIGURE 6. Threshold speed difference as a function of the spacing between the two gratings. The error bars represent ±1 SD of threshold.

grating under the conditions of our experiment. In this experiment the staircase adjusted the mean speed difference between distributions presented in the two intervals. Further, we decoupled the speed of the gratings from their direction by having half the gratings in an interval move to the left and the other half to the right. Specifically, we arranged the gratings so that alternate gratings in each interval moved in opposite directions. Feedback for the noise experiment was based on the distribution from which the gratings were drawn and not on their actual speed values. Since the distributions centered about these values could overlap, it is possible that some samples from the faster test speed distribution were indeed slower than samples from the slower reference speed distribution, especially when the difference between the reference and test speeds was small. In order to ensure that feedback was not adversely affecting the thresholds in this experiment, two of the four subjects (LS and ET) did the noise experiments without feedback.

Results. Figure 7(a) illustrates the data from four observers for the added noise experiment. Although absolute thresholds are elevated compared to the original experiment, they decrease with the number of gratings at a similar rate as the original experiment. This is better illustrated in the graph of Fig. 7(b) which plots relative thresholds (normalized to the threshold for one grating) averaged across our four observers, as a function of number, for the added-noise and no-noise cases. If we assume that observers used the same strategy in these two cases, then the fact that relative thresholds behave in a similar manner in these two cases suggests that correlation was not an important factor and that the speed estimates from individual grating patches in the original no-noise case are essentially independent.

Furthermore, the decrease in thresholds with number despite gratings moving in opposite directions argues against a global direction selective mechanism that sums inputs of similar direction selectivity. Such a mechanism would on average have no net input in this paradigm where equal numbers of gratings moved to the left and right. It could however be argued that thresholds would have decreased faster with number if all the gratings in an interval moved in the same direction. To test this, the aauthors performed the noise experiment with gratings moving in the same direction. The thresholds for these two observers for the same-direction condition (not shown) were indistinguishable from their thresholds for the case when alternate gratings moved in opposite directions. These results are consistent with speed discrimination being based on local mechanisms that process individual gratings rather than a global direction-sensitive mechanism that processes all the gratings in the display.

To summarize the results of Expt 1 and its controls, the area control experiment suggests that there is

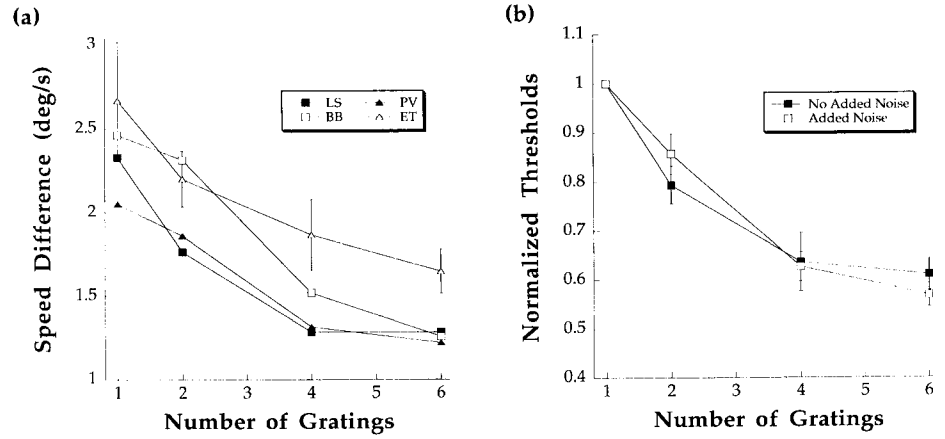

FIGURE 7. (a) Threshold speed difference as a function of the number of grating patches. The error bars represent ±1 SD of threshold. (b) Thresholds, normalized to that for one grating and averaged across observers, are plotted against the number of gratings. The error bars represent ±1 SE across observers. The open symbols are for the case when independent speed noise (from a Gaussian with a SD of 1.8 deg/sec) was added to each grating (data from (a)], and the solid symbols for the case when no noise was added (Fig. 3).

effectively a single speed estimate from each grating patch. In addition, the added-noise experiment shows that the estimates of speed from the multiple gratings are effectively independent. Taken together, these experiments suggest that the improvement in threshold with number of gratings is due to observers combining independent speed estimates, one from each grating patch.

Experiment 2

Uncertainty/search

Are the conclusions regarding the ability to combine information from multiple stimuli and the implication of independent samples applicable to speed discrimination in other paradigms? To examine this possibility we tested a different paradigm in which only a single grating in one of the intervals moved faster and observers were required to pick the interval with the faster grating. This paradigm is essentially an uncertainty experiment in which the observer has to monitor many channels, only one of which has the relevant stimulus (Graham & Nachmias, 1971; Davis, Kramer & Graham, 1983; Pelli, 1985). Increasing the number of irrelevant stimuli increases uncertainty. It is also similar to a search experiment in which observers have to detect the absence or presence of a target stimulus among several distractor stimuli (Treisman & Gelade, 1980). For instance, if the observer in the uncertainty/search task combined independent samples as suggested by Expt 1, and simply took the sum of the speed estimates from all the gratings in an interval, then the ability to detect the interval with the faster-moving grating would decrease as the number of slower-moving gratings was increased. Such a framework could predict the decrease in performance with number that is often observed in uncertainty and search experiments (Treisman & Gelade, 1980). If this simple framework can account for performance in our search task, it would be unnecessary to invoke the explanation that performance decreases with number because observers are limited in their ability to absorb information from multiple elements presented simultaneously. Moreover, our results in Expt 1 provide empirical proof that observers can combine information from at least four simultaneously presented gratings. Therefore, if the following search-like paradigm shows a decrease in performance with number up to four gratings, it is unlikely to be due to a limited input capacity.

Method. The spatial stimulus arrangement in the uncertainty (search) paradigm was identical to the original paradigm. The important difference was that only one of the n gratings in one of the intervals moved faster [Fig. 1(b)] and observers were required to detect the interval that contained the faster grating.

Results. Figure 8 plots threshold speed difference vs number of grating patches for our four observers. Thresholds increase with the number of gratings, increasing by about 50% from one to two gratings. This trend in the data is similar to results from both uncertainty and search experiments (Davis *et al.*, 1983; Treisman & Gelade, 1980). However the increase in thresholds with number cannot be attributed to an inability to combine information from multiple patches, which is the usual explanation for decrease in performance with number in search experiments. Experiment 1 argues strongly against this explanation as it shows that observers were indeed able to combine information from at least four gratings under the identical spatial configuration. In the following section we discuss simple models that predict the opposite trends of Expts 1 and 2.

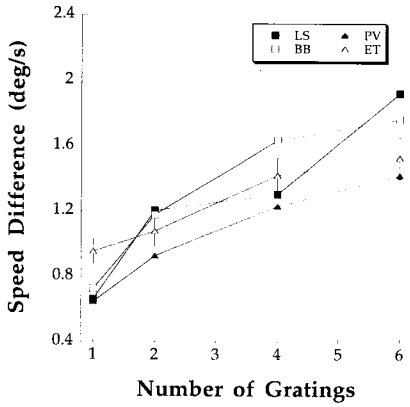

FIGURE 8. Threshold speed difference as a function of the number of grating patches for Expt 2. The different curves are for different observers as indicated in the figure legend. The error bars represent ±1 SD of the estimated threshold.

Models

Can the observed ability to use speed information from multiple stimuli be explained by the predictions of a simple decision model (Shaw, 1980; Graham, Kramer & Yager, 1987; Pavel *et al.*, 1992; Palmer *et al.*, 1993)? We assumed that all the gratings with the same speed had the same internal representation, which was a Gaussian distribution $F(x)$, with a density function $f(x)$. The mean of the Gaussian was equal to the speed of the grating and the SD was estimated from the data for one grating, as described below. Increasing the speed of the grating increased the mean of the Gaussian distribution proportionately, but the SD of the distribution was always the same. We assumed that n independent estimates of speed were available to the decision stage, one from each grating. We consider the predictions of the optimal model for each of the two paradigms, and compare these predictions to two other models, the maximum and the sum model.

The optimal model represents the performance of an ideal observer. This model calculates a likelihood ratio for each interval and picks the interval with the higher value of likelihood ratio. If there is a single speed estimate from each grating that is an independent sample from a Gaussian speed distribution, then for the paradigm of Expt 1, the optimal model predicts that thresholds will decrease by a factor of $\sqrt{n}$, where n is the number of grating patches (Green & Swets, 1966). The optimal model in this case is equivalent to a "sum"

model that sums the inputs from each interval and chooses the interval with the larger value. The dashed lines in Fig. 9 plot the predictions of the optimal (sum) model. Once again the predicted thresholds are plotted relative to that for one grating. The solid symbols in Fig. 9 show data from the original speed-discrimination experiment. Observed performance in this experiment does not improve as fast as the optimal model, indicating that observers are able to combine information, but do so less than optimally. An alternate explanation is that performance is close to optimal, but that part of the noise in the internal representation is correlated, causing performance to fall below optimal. If this hypothesis were correct, then adding large amounts of external speed noise would mask the effect of the correlated noise and cause thresholds to improve at rates closer to that predicted by the optimal model. The open symbols of Fig. 9 show data from our added noise experiment. Thresholds decrease at a similar rate as in the no-added noise case. This indicates that performance is less than optimal for reasons other than correlated noise.

We also consider the predictions of the maximum model, which chooses the interval that has the grating with the largest single speed sample. Our choice of this model is based on an observer's verbal report that, in the discrimination experiment, they sometimes resorted to choosing the interval that appeared to have the fastest grating. In Expt 1, the n gratings in one interval move at the same speed, s, and their internal representations are samples from a density function $f(x-s)$, while the n gratings in the other interval move at a faster speed, $s+\Delta s$, and their internal representations are samples from the density function $f(x-(s+\Delta s))$, where Δs is the speed difference between the two intervals. The probability of selecting the interval with the faster speed using a maximum model is

$$P(\Delta s)=\int_{-\infty}^{\infty} nf[x-(s+\Delta s)]F[x-(s+\Delta s)]^{n-1} \times F(x-s)^{n}\,dx. \quad (1a)$$

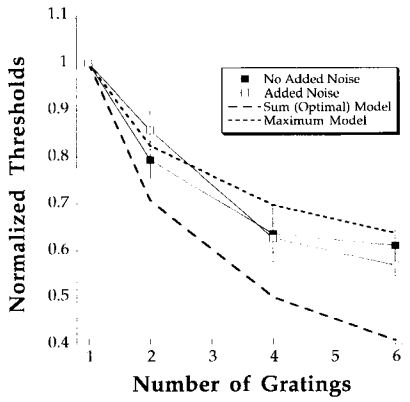

FIGURE 9. Thresholds for Expt 1 and the added noise control, normalized and averaged across observers, are plotted vs the number of gratings, along with the predictions of the optimal (sum) and maximum models.

With a change of variables, Equation 1(a) can be rewritten as

$$P(\Delta s)=\int_{-\infty}^{\infty} nf(x-\Delta s)F(x-\Delta s)^{n-1}F(x)^{n}\,dx. \quad (1b)$$

Similar equations have been derived by Palmer *et al.* (1993). The SD of the underlying Gaussian distribution was estimated by fitting equation (1b) to the data for a single grating, with $n=1$. The predicted psychometric curves for the other values of n were calculated assuming this value of SD. The dotted lines of Fig. 9 plot the predictions of the maximum model, and demonstrate that it is clearly suboptimal for the discrimination paradigm, as performance improves with number at a much slower rate than the optimal model. Observers' average thresholds improve at a rate close to, but slightly faster than, the maximum model.

We now consider the uncertainty experiment (Expt 2) to see how the trend of these data compare with optimal and sub-optimal models. For the uncertainty paradigm, the predicted performance of the optimal model is nearly indistinguishable from that of the maximum (Nolte & Jaarsma, 1967; Pelli, 1985). (Note the maximum model was far from optimal in Expt 1.) Therefore, we use the maximum model to estimate the predictions of the optimal model. Again, the maximum model picks the interval with the grating that has the fastest speed. In the uncertainty experiment the probability of choosing this interval is

$$P(\Delta s)=\int_{-\infty}^{\infty}[f(x-\Delta s)F(x)^{2n-1} + (n-1)f(x)F(x-\Delta s)F(x)^{2n-2}]\,dx. \quad (2)$$

For $n=1$, equations (1) and (2) are identical as is true of Expts 1 and 2. Figure 10 plots the predictions of the optimal (in this case, maximum) model as well as the normalized data averaged across all observers. Once again, the predicted thresholds are plotted relative to the prediction for one grating. Thresholds degrade with the number of gratings at a faster rate than the predictions of the optimal model. However, even an ideal observer with perfect knowledge of the distributions of the stimuli does worse with increasing number in this paradigm.

We also consider the "sum" model that adds speed estimates in each interval and chooses the interval with the larger value. This choice of model was based on some observers' verbal report that, for the uncertainty/search experiment, they based their decision on the mean speed in each interval, when they saw no clear outlier. The sum model predicts that thresholds will increase by a factor of $\sqrt{n}$, where n is the number of gratings in an interval. The dotted lines in Fig. 10 plot the predictions of the sum model. It is of interest to note that the sum model, which is optimal in Expt 1, is far from optimal in Expt 2. Observers' average thresholds are close to the predictions of the sum model.

In both Expts 1 and 2, performance falls short of optimal. For each case, we have outlined a non-optimal strategy that predicts the trend of the data better than

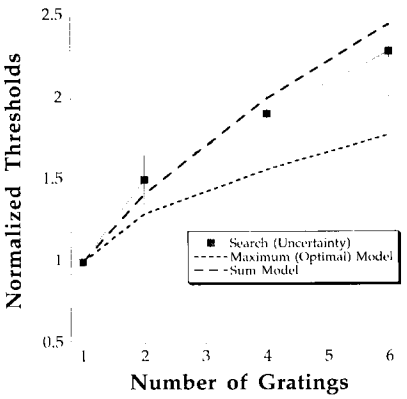

FIGURE 10. Thresholds for Expt 2, normalized and averaged across observers, are plotted vs the number of gratings, along with the predictions of the maximum (optimal) and sum models.

the optimal model. Alternatively, observers could have used an optimum decision strategy, preceded by a noisy stage that combines the multiple inputs. Another model that deserves consideration is one in which the ability to faithfully represent multiple inputs decreases with the number of inputs. We are currently examining these alternatives.

Experiment 3

Sequential vs simultaneous presentation

The decision models above assume that speed estimates from the different spatial locations and temporal intervals used in our experiment are equivalent. Therefore, these models predict that thresholds would be the same regardless of whether the discrimination is made across two spatial intervals (simultaneously) or two temporal intervals (sequentially).* As an independent test of this assumption, we explicitly compared speed discrimination in an experiment in which the two gratings to be discriminated were presented in either two temporal intervals, or in two spatial intervals.

Method. The sequential presentation was identical to the $n = 1$ condition of Expt 1 (or 2). In the simultaneous presentation, there was a single temporal interval with two gratings on opposite sides of fixation and observers were asked to decide whether the faster grating appeared to the left or the right of fixation. The gratings appeared at an eccentricity of 4 deg, at any pair of diametrically opposite locations except directly above and below the fixation point, as these locations were ambiguous regarding left right position.

Results. Figure 11 plots the data for all four observers in a task that measured speed discrimination between two gratings in simultaneous as well as sequential presentation. Speed differences for the sequential task are plotted vs those for the simultaneous task. The error bars in this plot represent 95% confidence limits on the estimate of threshold speed difference. These confidence limits are used to test the hypothesis that thresholds are similar in the sequential and simultaneous paradigms. The straight line of slope 1 and intercept 0 predicts where thresholds would lie if the performance in the two tasks were indistinguishable. The thresholds of three of our observers are consistent with this prediction. However, observer BB is significantly better at the sequential discrimination task. His poorer performance at the simultaneous task might reflect the fact that BB, while highly practiced at two-interval temporal forced-choice, has had little experience with two-spatial alternative tasks.

*Discriminating between two spatial intervals and two temporal intervals is equivalent only if each of the samples in these intervals is independent. If there is a correlation due to the samples being presented at the same instant of time or in the same spatial location, these two discriminations would no longer be equivalent.

DISCUSSION

Our data show different trends in the two speed discrimination paradigms that we studied. In the multiple-grating version of a two-interval speed-discrimination task, threshold speed differences decreased as the number of gratings in each interval increased. For the uncertainty or search paradigm in which observers chose the interval with the single grating that was moving faster, discrimination thresholds increased with the number of gratings. The opposite trends of thresholds in these two paradigms are predicted by a simple decision theory model that combines a single independent speed estimate from each grating. We have also shown that speed discrimination between two gratings is similar for sequential and simultaneous presentation. This result is also consistent with our decision model and shows that these two presentation modes are equivalent, at least for our stimulus conditions. Our experimental results taken together raise several issues: the ability to use information from multiple stimuli, the implications of the lack of an area effect, the effect of experimental paradigms, the nature of the underlying neural mechanisms,

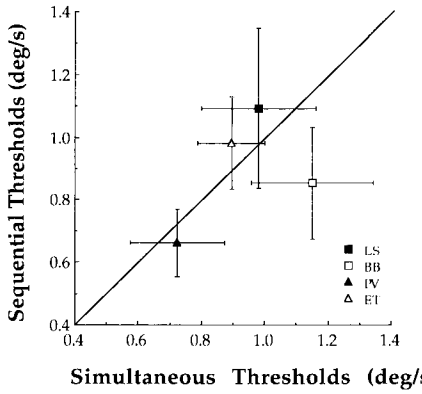

FIGURE 11. Threshold speed difference for the sequential presentation case are plotted vs the simultaneous presentation case (Expt 3). The error bars represent 95% confidence intervals. The straight line has a slope of 1, and a y-intercept of 0, and predicts where thresholds would lie if the performance in the two tasks was indistinguishable.

digms, the nature of the underlying neural mechanisms, and the implications for visual search. We will consider each of these in turn.

Combining information from multiple stimuli

The results of Expt 1 show that observers are able to combine information from multiple stimuli distributed across the visual field. These results are consistent with studies of target redundancy that showed that the probability of detecting the target increases with the number of target stimuli (Ericksen, 1966; Santee & Egeth, 1982). Two of our control experiments show that the improvement in speed discrimination threshold with number is not simply due to the increased probability of a grating appearing in a particular location, nor due to increased stimulus area with number of gratings. The results of a third control experiment in which independent speed noise was added to each grating are consistent with the speed estimates from each grating patch being independent. We considered the predictions of decision models that combine independent samples the optimum model, as well as the sum and maximum models. For Expt 1, the sum model is equivalent to the optimal model, whereas the maximum model uses a sub-optimal strategy. Both models predict that thresholds decrease with the number of gratings. While the data are close to the predictions of the sub-optimal maximum model, it is also possible that observers use an imperfect sum or average rule consistent with Watamaniuk and Duchon's (1992) study of speed discrimination using random dots moving at a range of speeds.

In Expt 2, thresholds for detecting the single faster moving grating increase with the number of gratings. For this experiment the maximum model is indistinguishable from optimal, whereas the sum model is clearly sub-optimal. Both models predict an increase in threshold with number. However, the data are better predicted by the sub-optimal sum model than the optimum model. The results of both experiments are consistent with observers combining independent samples across space. It would be interesting to see if this ability and the model predictions apply to the combination of signals other than speed measures from moving gratings.

Implications of the lack of an area effect

The observed lack of threshold improvement with increasing grating area appears to conflict with the results of classical psychophysical experiments on grating summation (Robson & Graham, 1981). The lack of an area effect in our experiments could arise from the stimuli being presented well above detection threshold (we used a contrast of 50%), and more than two cycles being visible even for the smallest grating that we used. It has been shown that for contrasts larger than about 6%, the apparent contrast of a grating windowed by a Gaussian with a space constant of 0.5 cycle is not significantly different from a full-field grating (Cannon & Fullenkamp, 1988; Swanson, Wilson & Geise, 1984; Takahashi & Ejima, 1984).

From the point of view of our simple decision model, the fact that threshold does not improve with the area of a single patch is consistent with each grating patch effectively providing only one speed estimate to the decision stage, regardless of how big the patch is. The single speed estimate per object is also consistent with the results of He and Nakayama (1994a, b). Their data for texture discrimination and apparent motion tasks show that when the binocular disparity of stimuli was manipulated so that their surface representation changed, while the output of "early filters" was left unchanged, observers were unable to ignore surface shape. They therefore conclude that the representation of surfaces and object boundaries occurs at a relatively early stage. If indeed object segmentation occurs early, it is likely that the suprathreshold gratings that we use in our experiments are represented as distinct objects and that a single speed estimate is assigned to each. Another mechanism that is consistent with our results is one in which object segmentation does not occur explicitly, but that the single speed estimate results from taking a local maximum of the speed in a "region". One might further hypothesize that the "region" or object is demarcated by areas of no optic flow. These mechanisms are admittedly speculative, but we offer them as possible ways to go from multiple estimates of speed per object to a single estimate.

Effect of experimental paradigm

As we have demonstrated, the difference in results between the multiple-grating speed discrimination and the uncertainty paradigm can be understood in terms of simple decision models. The paradigms of Expts 1 and 2 were quite similar: they had the same stimuli, the same spatial configuration and required the same response. The single difference, that Expt 1 had multiple signal (target) gratings whereas Expt 2 always had only one signal grating, caused opposite trends in both measured and predicted thresholds as the number of elements was increased. The different results from these two paradigms are not because Expt 1 required a comparison of gratings between two sequential temporal intervals, while Expt 2 required a comparison of gratings presented simultaneously, within the same temporal interval. A signal detection model that assumes that the speed estimates from the different spatial locations and temporal intervals are equivalent, predicts the same thresholds for sequential and simultaneous presentations. This assumption is borne out by the results of Expt 3. Most of our observers showed no difference in thresholds between these two conditions. Thus, Expt 3, provides further support for the independence of spatial and temporal samples. This result also suggests that the different reports of the effect of contrast and spatial frequency on speed discrimination are unlikely to be due to simultaneous vs sequential stimulus presentation *per se* (Diener *et al.*, 1976; Thompson, 1982; McKee *et al.*, 1986; Ferrara & Wilson, 1991; Smith & Edgar, 1990; Stone & Thompson, 1992; Hawken *et al.*, 1994).

Constraints on underlying neural mechanisms

It could be argued that performance in our experiments was mediated by a single, direction-selective mechanism with a large receptive field that covered the entire stimulus. However, the results from the added-noise experiment with gratings moving in opposite directions, strongly argue against such a possibility. The diameter of the average receptive field in the primate middle temporal area (MT) is approximately equal to the eccentricity of the receptive field, with the SD from that size being about one-third (Albright & Desimone, 1987). If humans have receptive fields of comparable size at this eccentricity, then the receptive field diameter at an eccentricity of 4 deg is about 4 ± 1.3 deg. Anderson and Burr's (1989, 1991) estimate of the "psychophysical" receptive field of a motion unit at this eccentricity is a Gabor with a SD of 0.3 deg. As the closest center-to-center spacing of the gratings in our experiments was 4 deg and the SD of the Gaussian window 0.4 deg, it is unlikely that the receptive field of an MT cell or psychophysical motion unit overlapped more than one complete grating patch. These estimates of functional receptive field size also support the argument that speed discrimination in our experimental configuration was based on local independent samples of speed.

Furthermore, the data from the noise experiment with gratings moving in opposite directions (Fig. 7), argue against discrimination performance being mediated by simple summing within units with large receptive fields such as those in the primate medial superior temporal area (Tanaka, Hikosaka, Saito, Yukie, Fukada & Iwai, 1986). However, it is possible that units sensitive to special combinations of motion played a role. The experiment with alternate gratings moving in opposite directions resulted (randomly) in some stimulus presentations in which the gratings moved toward each other, away from each other, or in shearing motion, and therefore could have preferentially activated higher-order motion units tuned to compression, expansion and shearing motions (Koenderink, 1986; Tanaka *et al.*, 1986; Anderson, Snowden, Treue & Graziano, 1990; Duffy & Wurtz, 1991; Lagae, Maes, Raiguel, Xiao & Orban, 1994). As these configurations occurred randomly, it is unlikely that the same type of stimulus occurred in the two intervals of each trial. Given these stimulus conditions, it is difficult to explain our results using a decision based on a comparison of different types of higher-order motion units.

Implications for search tasks

The underlying assumption in search tasks is that performance is subject to a bottleneck in visual processing (Broadbent, 1958; Neisser, 1967). If the display is brief and the number of distractor elements is increased beyond the capacity of the bottleneck, then a smaller fraction of the elements is processed, resulting in a lower probability correct. The decrement of performance with increasing number of elements in search tasks has therefore been attributed to the limited amount of information that can be processed in a brief display. Our uncertainty/search experiment also shows a decrement in performance when the number of irrelevant grating patches is increased. However, this decrement cannot simply be due to an inability to absorb information from multiple gratings, as our first experiment indicates that observers are able to combine information from at least four discrete grating patches in exactly the same spatial arrangement. In addition, this trend is predicted by a simple decision model that does not assume any processing limit, confirming other studies that have described visual search performance in terms of decision theory models (Pavel *et al.*, 1992; Palmer *et al.*, 1993; Verghese & Nakayama, 1994). These arguments, the first experimental and the second model-based, raise the question of what role processing limits play in other examples of visual search.

The thresholds in our search experiment increase at a faster rate than predicted by an ideal observer. Therefore it might be argued that the additional decrement in performance is due to a limit on the number of gratings that are processed. If this were true it would be hard to explain why thresholds decrease with increasing number in Expt 1. An alternate explanation for why observed performance degrades at a faster rate than the optimum model observer is because observers use a non-optimal strategy. Our data and the results of our preliminary modeling suggest that processing limits need not be invoked to explain the decrement in search performance; non-optimal combination of stimulus information is a plausible alternative that deserves further evaluation.

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